HL Paper 2

The graph of $y = \ln(5x + 10)$ is obtained from the graph of $y = \ln x$ by a translation of a units in the direction of the x-axis followed by a translation of b units in the direction of the y-axis.

a. Find the value of a and the value of b. [4]

b. The region bounded by the graph of $y = \ln(5x + 10)$, the x-axis and the lines $x = \mathrm{e}$ and $x = 2\mathrm{e}$, is rotated through 2π radians about the x-axis. Find the volume generated. [2]

Markscheme

a. **EITHER**

$y = \ln(x - a) + b = \ln(5x + 10)$ ***(M1)***

$y = \ln(x - a) + \ln c = \ln(5x + 10)$

$y = \ln(c(x - a)) = \ln(5x + 10)$ ***(M1)***

OR

$y = \ln(5x + 10) = \ln(5(x + 2))$ ***(M1)***

$y = \ln(5) + \ln(x + 2)$ ***(M1)***

THEN

$a = -2,\ b = \ln 5$ ***A1A1***

Note: Accept graphical approaches.

Note: Accept $a = 2,\ b = 1.61$

[4 marks]

b. $V = \pi \int_e^{2e} [\ln(5x + 10)]^2 \mathrm{d}x$ ***(M1)***

$= 99.2$ ***A1***

[2 marks]

Total [6 marks]

Examiners report

a. [N/A]

b. [N/A]

Consider $p(x) = 3x^3 + ax + 5a, \quad a \in \mathbb{R}$.

The polynomial $p(x)$ leaves a remainder of -7 when divided by $(x - a)$.

Show that only one value of a satisfies the above condition and state its value.

Markscheme

using $p(a) = -7$ to obtain $3a^3 + a^2 + 5a + 7 = 0$ ***M1A1***

$(a + 1)(3a^3 - 2a + 7) = 0$ ***(M1)(A1)***

Note: Award ***M1*** for a cubic graph with correct shape and ***A1*** for clearly showing that the above cubic crosses the horizontal axis at $(-1,\ 0)$ only.

$a = -1$ ***A1***

EITHER

showing that $3a^2 - 2a + 7 = 0$ has no real (two complex) solutions for a ***R1***

OR

showing that $3a^3 + a^2 + 5a + 7 = 0$ has one real (and two complex) solutions for a ***R1***

Note: Award ***R1*** for solutions that make specific reference to an appropriate graph.

[6 marks]

Examiners report

A large number of candidates, either by graphical (mostly) or algebraic or via use of a GDC solver, were able to readily obtain $a = -1$. Most candidates who were awarded full marks however, made specific reference to an appropriate graph. Only a small percentage of candidates used the discriminant to justify that only one value of a satisfied the required condition. A number of candidates erroneously obtained $3a^3 + a^2 + 5a - 7 = 0$ or equivalent rather than $3a^3 + a^2 + 5a + 7 = 0$.

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a. Find $\int x\sec^2 x\mathrm{d}x$. [4]

b. Determine the value of m if $\int_0^m x\sec^2 x\mathrm{d}x = 0.5$, where $m > 0$. [2]

Markscheme

a. $\int x\sec^2 x\mathrm{d}x = x\tan x - \int 1 \times \tan x\mathrm{d}x$ ***M1A1***

$= x\tan x + \ln|\cos x|(+c) \ (= x\tan x - \ln|\sec x|(+c))$ ***M1A1***

[4 marks]

b. attempting to solve an appropriate equation *eg* $m\tan m + \ln(\cos m) = 0.5$ ***(M1)***

$m = 0.822$ ***A1***

Note: Award ***A1*** if $m = 0.822$ is specified with other positive solutions.

[2 marks]

Examiners report

a. In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of $\tan x$. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

b. In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of $\tan x$. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

It is given that $f(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$ where a and b are positive integers.

a. Given that $x^2 - 1$ is a factor of $f(x)$ find the value of a and the value of b. [4]

b. Factorize $f(x)$ into a product of linear factors. [3]

c. Sketch the graph of $y = f(x)$, labelling the maximum and minimum points and the x and y intercepts. [3]

d. Using your graph state the range of values of c for which $f(x) = c$ has exactly two distinct real roots. [3]

Markscheme

a. $g(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$

$g(1) = 0 \Rightarrow a + b = 8$ ***M1A1***

$g(-1) = 0 \Rightarrow -a + b = -6$ ***A1***

$\Rightarrow a = 7,\ b = 1$ ***A1***

[4 marks]

b. $3x^4 + 7x^3 + x^2 - 7x - 4 = (x^2 - 1)(px^2 + qx + r)$

attempt to equate coefficients ***(M1)***

$p = 3,\ q = 7,\ r = 4$ ***(A1)***

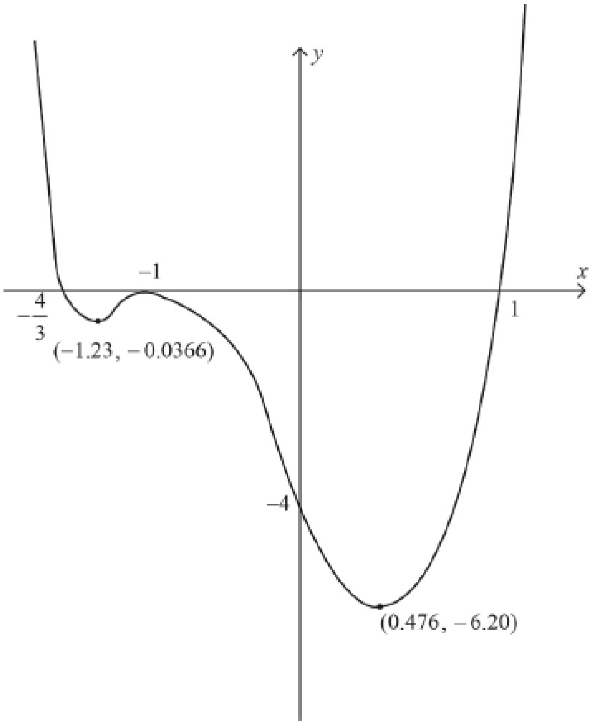
$3x^4 + 7x^3 + x^2 - 7x - 4 = (x^2 - 1)(3x^2 + 7x + 4)$

$= (x - 1)(x + 1)^2(3x + 4)$ ***A1***

Note: Accept any equivalent valid method.

[3 marks]

c.



A1 for correct shape (*ie* with correct number of max/min points)

A1 for correct x and y intercepts

A1 for correct maximum and minimum points

[3 marks]

d. $c > 0$ ***A1***

$-6.20 < c < -0.0366$ ***A1A1***

Note: Award ***A1*** for correct end points and ***A1*** for correct inequalities.

Note: If the candidate has misdrawn the graph and omitted the first minimum point, the maximum mark that may be awarded is ***A1FTA0A0*** for $c > -6.20$ seen.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The function $f(x) = 3\sin x + 4\cos x$ is defined for $0 < x < 2\pi$.

a. Write down the coordinates of the minimum point on the graph of f. [1]

b. The points $\mathrm{P}(p,\ 3)$ and $\mathrm{Q}(q,\ 3)$, $q > p$, lie on the graph of $y = f(x)$. [2]

Find p and q .

c. Find the coordinates of the point, on $y = f(x)$, where the gradient of the graph is 3. [4]

d. Find the coordinates of the point of intersection of the normals to the graph at the points P and Q. [7]

Markscheme

a. $(3.79, -5)$ ***A1***

[1 mark]

b. $p = 1.57$ or $\frac{\pi}{2}$, $q = 6.00$ ***A1A1***

[2 marks]

c. $f'(x) = 3\cos x - 4\sin x$ ***(M1)(A1)***

$3\cos x - 4\sin x = 3 \Rightarrow x = 4.43...$ ***(A1)***

$(y = -4)$ ***A1***

Coordinates are $(4.43, -4)$

[4 marks]

d. $m_{\text{normal}} = \frac{1}{m_{\text{tangent}}}$ ***(M1)***

gradient at P is -4 so gradient of normal at P is $\frac{1}{4}$ ***(A1)***

gradient at Q is 4 so gradient of normal at Q is $-\frac{1}{4}$ ***(A1)***

equation of normal at P is $y - 3 = \frac{1}{4}(x - 1.570...)$ (or $y = 0.25x + 2.60...$) ***(M1)***

equation of normal at Q is $y - 3 = \frac{1}{4}(x - 5.999...)$ (or $y = -0.25x + \underbrace{4.499...}$) ***(M1)***

Note: Award the previous two ***M1*** even if the gradients are incorrect in $y - b = m(x - a)$ where (a, b) are coordinates of P and Q (or in $y = mx + c$ with c determined using coordinates of P and Q.

intersect at $(3.79,\ 3.55)$ ***A1A1***

Note: Award ***N2*** for 3.79 without other working.

[7 marks]

Examiners report

a. Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

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Show that the quadratic equation $x^2 - (5-k)x - (k+2) = 0$ has two distinct real roots for all real values of k .

Markscheme

$\Delta = (5-k)^2 + 4(k+2)$ ***M1A1***
$= k^2 - 6k + 33$ ***(A1)***
$= (k-3)^2 + 24$ which is positive for all k ***R1***
Note: Accept analytical, graphical or other correct methods. In all cases only award ***R1*** if a reason is given in words or graphically. Award ***M1A1A0R1*** if mistakes are made in the simplification but the argument given is correct.

[4 marks]

Examiners report

Overall the question was pretty well answered but some candidates seemed to have mixed up the terms determinant with discriminant. In some cases a lack of quality mathematical reasoning and understanding of the discriminant was evident. Many worked with the quadratic formula rather than just the discriminant, conveying a lack of understanding of the strategy required. Errors in algebraic simplification (expanding terms involving negative signs) prevented many candidates from scoring well in this question. Many candidates were not able to give a clear reason why the quadratic has always two distinct real solutions; in some cases a vague explanation was given, often referring to a graph which was not sketched.

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a. Express $x^2 + 4x - 2$ in the form $(x+a)^2 + b$ where $a,\ b \in \mathbb{Z}$. [2]

b. If $f(x) = x + 2$ and $(g \circ f)(x) = x^2 + 4x - 2$ write down $g(x)$. [2]

Markscheme

a. $(x+2)^2 - 6$ ***A1A1***

[2 marks]

b. $(g \circ f)(x) = (x+2)^2 - 6$ ***(M1)***

$\Rightarrow g(x) = x^2 - 6$ ***A1***

[2 marks]

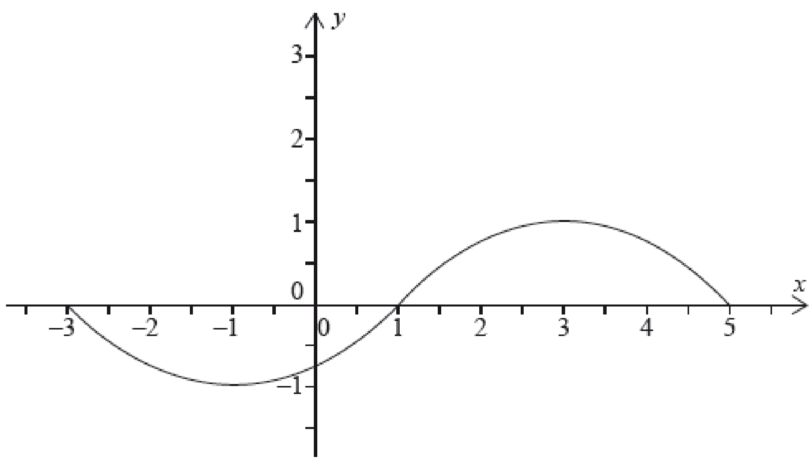
Examiners report

a. Well done by most candidates.

b. Well done by most candidates. Those students who lost marks on this question tended to do so in part (b), seemingly through misinterpreting the question.

The following graph represents a function $y = f(x)$, where $-3 \le x \le 5$.

The function has a maximum at $(3,\ 1)$ and a minimum at $(-1,\ -1)$.



a. The functions u and v are defined as $u(x) = x - 3,\ v(x) = 2x$ where $x \in \mathbb{R}$. [7]

(i) State the range of the function $u \circ f$.

(ii) State the range of the function $u \circ v \circ f$.

(iii) Find the largest possible domain of the function $f \circ v \circ u$.

b. (i) Explain why f does not have an inverse. [6]

(ii) The domain of f is restricted to define a function g so that it has an inverse g^{-1}.

State the largest possible domain of g.

(iii) Sketch a graph of $y = g^{-1}(x)$, showing clearly the y-intercept and stating the coordinates of the endpoints.

c. Consider the function defined by $h(x) = \frac{2x-5}{x+d}$, $x \ne -d$ and $d \in \mathbb{R}$. [8]

(i) Find an expression for the inverse function $h^{-1}(x)$.

(ii) Find the value of d such that h is a self-inverse function.

For this value of d, there is a function k such that $h \circ k(x) = \frac{2x}{x+1},\ x \ne -1$.

(iii) Find $k(x)$.

Markscheme

a. **Note:** For Q12(a) (i) – (iii) and (b) (ii), award ***A1*** for correct endpoints and, if correct, award ***A1*** for a closed interval.

Further, award ***A1A0*** for one correct endpoint and a closed interval.

(i) $-4 \le y \le -2$ ***A1A1***

(ii) $-5 \le y \le -1$ ***A1A1***

(iii) $-3 \le 2x - 6 \le 5$ ***(M1)***

Note: Award ***M1*** for $f(2x-6)$.

$3 \le 2x \le 11$

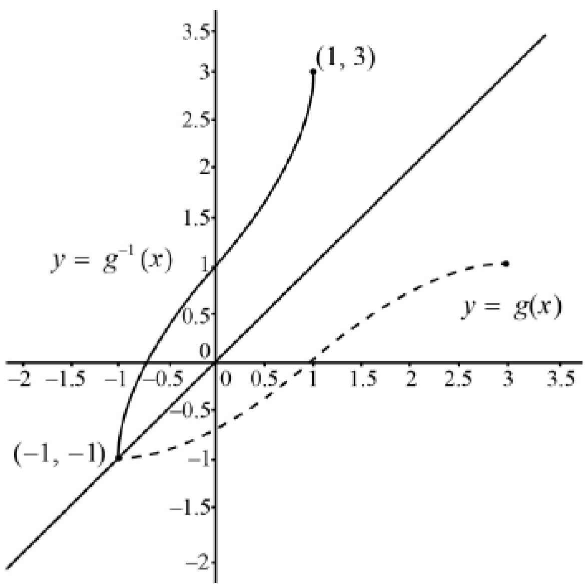
$\frac{3}{2} \le x \le \frac{11}{2}$ ***A1A1***

[7 marks]

b. (i) any valid argument *eg* f is not one to one, f is many to one, fails horizontal line test, not injective ***R1***

(ii) largest domain for the function $g(x)$ to have an inverse is $[-1,\ 3]$ ***A1A1***

(iii)



y-intercept indicated (coordinates not required) ***A1***

correct shape ***A1***

coordinates of end points $(1,\ 3)$ and $(-1,\ -1)$ ***A1***

Note: Do not award any of the above marks for a graph that is not one to one.

[6 marks]

c. (i) $y = \frac{2x-5}{x+d}$

$(x+d)y = 2x - 5$ ***M1***

Note: Award ***M1*** for attempting to rearrange x and y in a linear expression.

$x(y-2) = -dy - 5$ ***(A1)***

$x = \frac{-dy-5}{y-2}$ ***(A1)***

Note: x and y can be interchanged at any stage

$h^{-1}(x) = \frac{-dx-5}{x-2}$ ***A1***

Note: Award ***A1*** only if $h^{-1}(x)$ is seen.

(ii) self Inverse $\Rightarrow h(x) = h^{-1}(x)$

$\frac{2x-5}{x+d} \equiv \frac{-dx-5}{x-2}$ ***(M1)***

$d = -2$ ***A1***

(iii) **METHOD 1**

$\frac{2k(x)-5}{k(x)-2} = \frac{2x}{x+1}$ ***(M1)***

$k(x) = \frac{x+5}{2}$ ***A1***

METHOD 2

$h^{-1}\left(\frac{2x}{x+1}\right) = \frac{2\left(\frac{2x}{x+1}\right)-5}{\frac{2x}{x+1}-2}$ ***(M1)***

$k(x) = \frac{x+5}{2}$ ***A1***

[8 marks]

Total [21 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The function f has inverse f^{-1} and derivative $f'(x)$ for all $x \in \mathbb{R}$. For all functions with these properties you are given the result that for $a \in \mathbb{R}$ with $b = f(a)$ and $f'(a) \neq 0$

$$(f^{-1})'(b) = \frac{1}{f'(a)}.$$

a. Verify that this is true for $f(x) = x^3 + 1$ at $x = 2$. [6]

b. Given that $g(x) = xe^{x^2}$, show that $g'(x) > 0$ for all values of x. [3]

c. Using the result given at the start of the question, find the value of the gradient function of $y = g^{-1}(x)$ at $x = 2$. [4]

d. (i) With f and g as defined in parts (a) and (b), solve $g \circ f(x) = 2$. [6]

(ii) Let $h(x) = (g \circ f)^{-1}(x)$. Find $h'(2)$.

Markscheme

a. $f(2) = 9$ ***(A1)***

$f^{-1}(x) = (x-1)^{\frac{1}{3}}$ ***A1***

$(f^{-1})'(x) = \frac{1}{3}(x-1)^{-\frac{2}{3}}$ ***(M1)***

$(f^{-1})'(9) = \frac{1}{12}$ ***A1***

$f'(x) = 3x^2$ ***(M1)***

$\frac{1}{f'(2)} = \frac{1}{3\times 4} = \frac{1}{12}$ ***A1***

Note: The last ***M1*** and ***A1*** are independent of previous marks.

[6 marks]

b. $g'(x) = \mathrm{e}^{x^2} + 2x^2\mathrm{e}^{x^2}$ ***M1A1***

$g'(x) > 0$ as each part is positive ***R1***

[3 marks]

c. to find the *x*-coordinate on $y = g(x)$ solve

$2 = x\mathrm{e}^{x^2}$ ***(M1)***

$x = 0.89605022078\ldots$ ***(A1)***

gradient $= (g^{-1})'(2) = \frac{1}{g'(0.896\ldots)}$ ***(M1)***

$= \frac{1}{\mathrm{e}^{(0.896\ldots)^2}\left(1+2\times(0.896\ldots)^2\right)} = 0.172$ to 3sf ***A1***

(using the $\frac{\mathrm{d}y}{\mathrm{d}x}$ function on gdc $g'(0.896\ldots) = 5.7716028\ldots$

$\frac{1}{g'(0.896\ldots)} = 0.173$

[4 marks]

d. (i) $(x^3+1)\mathrm{e}^{(x^3+1)^2} = 2$ ***A1***

$x = -0.470191\ldots$ ***A1***

(ii) **METHOD 1**

$(g \circ f)'(x) = 3x^2\mathrm{e}^{(x^3+1)^2}\left(2(x^3+1)^2+1\right)$ ***(M1)(A1)***

$(g \circ f)'(-0.470191\ldots) = 3.85755\ldots$ ***(A1)***

$h'(2) = \frac{1}{3.85755\ldots} = 0.259\ (232\ldots)$ ***A1***

Note: The solution can be found without the student obtaining the explicit form of the composite function.

METHOD 2

$h(x) = (f^{-1} \circ g^{-1})(x)$ ***A1***

$h'(x) = (f^{-1})'\left(g^{-1}(x)\right) \times (g^{-1})'(x)$ ***M1***

$= \frac{1}{3}\left(g^{-1}(x)-1\right)^{-\frac{2}{3}} \times (g^{-1})'(x)$ ***M1***

$h'(2) = \frac{1}{3}\left(g^{-1}(2)-1\right)^{-\frac{2}{3}} \times (g^{-1})'(2)$

$= \frac{1}{3}(0.89605\ldots - 1)^{-\frac{2}{3}} \times 0.171933\ldots$

$= 0.259\ (232\ldots)$ ***A1*** ***N4***

[6 marks]

Examiners report

a. There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification.

b. There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification.

c. Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

d. Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

Let $f(x) = x(x+2)^6$.

a. Solve the inequality $f(x) > x$. [5]

b. Find $\int f(x)\mathrm{d}x$. [5]

Markscheme

a. **METHOD 1**

sketch showing where the lines cross or zeros of $y = x(x+2)^6 - x$ ***(M1)***

$x = 0$ ***(A1)***

$x = -1$ and $x = -3$ ***(A1)***

the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 2

separating into two cases $x > 0$ and $x < 0$ ***(M1)***

if $x > 0$ then $(x+2)^6 > 1 \Rightarrow$ always true ***(M1)***

if $x < 0$ then $(x+2)^6 < 1 \Rightarrow -3 < x < -1$ ***(M1)***

so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 3

$f(x) = x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x$ ***(A1)***

solutions to $x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 63x = 0$ are ***(M1)***

$x = 0,\ x = -1$ and $x = -3$ ***(A1)***

so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 4

$f(x) = x$ when $x(x+2)^6 = x$

either $x = 0$ or $(x+2)^6 = 1$ ***(A1)*** if $(x+2)^6 = 1$ then $x+2 = \pm 1$ so $x = -1$ or $x = -3$ ***(M1)(A1)***

the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

[5 marks]

b. **METHOD 1** (by substitution)

substituting $u = x+2$ ***(M1)***

$\mathrm{d}u = \mathrm{d}x$

$\int (u-2)u^6 \mathrm{d}u$ ***M1A1***

$= \frac{1}{8}u^8 - \frac{2}{7}u^7 (+c)$ ***(A1)***

$= \frac{1}{8}(x+2)^8 - \frac{2}{7}(x+2)^7 (+c)$ ***A1***

METHOD 2 (by parts)

$u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1,\ \frac{\mathrm{d}v}{\mathrm{d}x} = (x+2)^6 \Rightarrow v = \frac{1}{7}(x+2)^7$ ***(M1)(A1)***

$\int x(x+2)^6 \mathrm{d}x = \frac{1}{7}x(x+2)^7 - \frac{1}{7}\int (x+2)^7 \mathrm{d}x$ ***M1***

$= \frac{1}{7}x(x+2)^7 - \frac{1}{56}(x+2)^8 (+c)$ ***A1A1***

METHOD 3 (by expansion)

$\int f(x)\mathrm{d}x = \int \left(x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x\right) \mathrm{d}x$ ***M1A1***

$= \frac{1}{8}x^8 + \frac{12}{7}x^7 + 10x^6 + 32x^5 + 60x^4 + 64x^3 + 32x^2 (+c)$ ***M1A2***

Note: Award ***M1A1*** if at least four terms are correct.

[5 marks]

Examiners report

a. [N/A]

b. [N/A]

One root of the equation $x^2 + ax + b = 0$ is $2 + 3\mathrm{i}$ where $a,\ b \in \mathbb{R}$. Find the value of a and the value of b.

Markscheme

METHOD 1

substituting

$-5 + 12\mathrm{i} + a(2 + 3\mathrm{i}) + b = 0$ ***(A1)***

equating real or imaginary parts ***(M1)***

$12 + 3a = 0 \Rightarrow a = -4$ ***A1***

$-5 + 2a + b = 0 \Rightarrow b = 13$ ***A1***

METHOD 2

other root is $2 - 3\mathrm{i}$ ***(A1)***

considering either the sum or product of roots or multiplying factors ***(M1)***

$4 = -a$ (sum of roots) so $a = -4$ ***A1***

$13 = b$ (product of roots) ***A1***

[4 marks]

Examiners report

[N/A]

The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of $(x - 1)$, $(x - 2)$ and $(x - 3)$.

Find the values of p, q and r.

Markscheme

METHOD 1

substitute each of x = 1,2 and 3 into the quartic and equate to zero ***(M1)***

$p + q + r = -7$

$4p + 2q + r = -11$ or equivalent ***(A2)***

$9p + 3q + r = -29$

Note: Award ***A2*** for all three equations correct, ***A1*** for two correct.

attempting to solve the system of equations ***(M1)***

p = –7, q = 17, r = –17 ***A1***

Note: Only award ***M1*** when some numerical values are found when solving algebraically or using GDC.

METHOD 2

attempt to find fourth factor ***(M1)***

$(x - 1)$ ***A1***

attempt to expand $(x - 1)^2 (x - 2) (x - 3)$ ***M1***

$x^4 - 7x^3 + 17x^2 - 17x + 6$ (p = –7, q = 17, r = –17) ***A2***

Note: Award ***A2*** for all three values correct, ***A1*** for two correct.

Note: Accept long / synthetic division.

[5 marks]

Examiners report

[N/A]

A function f is defined by $f(x) = x^3 + \mathrm{e}^x + 1,\ x \in \mathbb{R}$. By considering $f'(x)$ determine whether f is a one-to-one or a many-to-one function.

Markscheme

$f'(x) = 3x^2 + \mathrm{e}^x$ ***A1***

Note: Accept labelled diagram showing the graph $y = f'(x)$ above the *x*-axis;

do not accept unlabelled graphs nor graph of $y = f(x)$.

EITHER

this is always > 0 ***R1***

so the function is (strictly) increasing ***R1***

and thus $1 - 1$ ***A1***

OR

this is always $> 0 \quad (\text{accept } \neq 0)$ ***R1***

so there are no turning points ***R1***

and thus $1 - 1$ ***A1***

Note: ***A1*** is dependent on the first ***R1***.

[4 marks]

Examiners report

The differentiation was normally completed correctly, but then a large number did not realise what was required to determine the type of the original function. Most candidates scored 1/4 and wrote explanations that showed little or no understanding of the relation between first derivative and the given function. For example, it was common to see comments about horizontal and vertical line tests but applied to the incorrect function.In term of mathematical language, it was noted that candidates used many terms incorrectly showing no knowledge of the meaning of terms like ‘parabola’, ‘even’ or ‘odd’ (or no idea about these concepts).

When carpet is manufactured, small faults occur at random. The number of faults in Premium carpets can be modelled by a Poisson distribution with mean 0.5 faults per 20 m^2. Mr Jones chooses Premium carpets to replace the carpets in his office building. The office building has 10 rooms, each with the area of 80 m^2.

a. Find the probability that the carpet laid in the first room has fewer than three faults. [3]

b. Find the probability that exactly seven rooms will have fewer than three faults in the carpet. [3]

Markscheme

a. $\lambda = 4 \times 0.5$ ***(M1)***

$\lambda = 2$ ***(A1)***

$\mathrm{P}(X \leqslant 2) = 0.677$ ***A1***

[3 marks]

b. $Y \sim B(10,\ 0{,}677)$ ***(M1)(A1)***

$\mathrm{P}(Y = 7) = 0.263$ ***A1***

Note: Award ***M1*** for clear recognition of binomial distribution.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

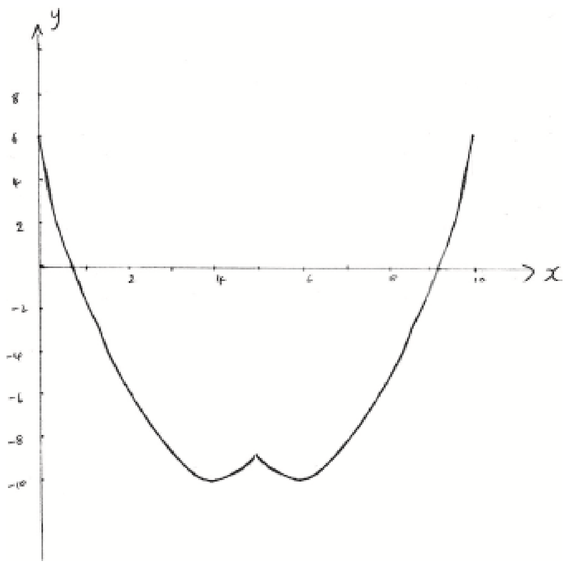
---

a. Sketch the graph of $y = (x-5)^2 - 2\left|x-5\right| - 9,\ \text{for } 0 \le x \le 10.$ [3]

b. Hence, or otherwise, solve the equation $(x-5)^2 - 2\left|x-5\right| - 9 = 0.$ [2]

Markscheme

a.



general shape including \[(\) minimums, cusp ***A1A1***

correct domain and symmetrical about the middle $(x = 5)$ ***A1***

[3 marks]

b. $x = 9.16 \quad \text{or} \quad x = 0.838$ ***A1A1***

[2 marks]

Total [5 marks]

Examiners report

a. [N/A]
b. [N/A]

The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} 0,\ x<0 \\ \frac{\sin x}{4},\ 0\le x\le\pi \\ a(x-\pi),\ \pi<x\le 2\pi \\ 0,\ 2\pi<x \end{cases}.$$

a. Sketch the graph $y=f(x)$. [2]

b. Find $\mathrm{P}(X\le\pi)$. [2]

c. Show that $a=\frac{1}{\pi^2}$. [3]

d. Write down the median of X. [1]

e. Calculate the mean of X. [3]

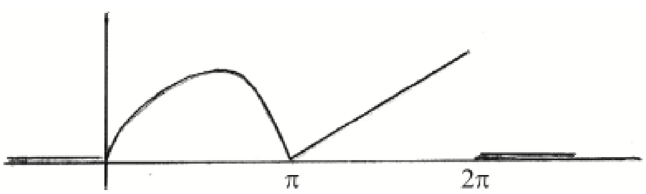
f. Calculate the variance of X. [3]

g. Find $\mathrm{P}\left(\frac{\pi}{2}\le X\le\frac{3\pi}{2}\right)$. [2]

h. Given that $\frac{\pi}{2}\le X\le\frac{3\pi}{2}$ find the probability that $\pi\le X\le 2\pi$. [4]

Markscheme

a.



Award ***A1*** for sine curve from 0 to π, award ***A1*** for straight line from π to 2π ***A1A1***

[2 marks]

b. $\int_0^\pi \frac{\sin x}{4}\mathrm{d}x=\frac{1}{2}$ ***(M1)A1***

[2 marks]

c. **METHOD 1**

require $\frac{1}{2}+\int_\pi^{2\pi} a(x-\pi)\mathrm{d}x=1$ ***(M1)***

$\Rightarrow \frac{1}{2}+a\left[\frac{(x-\pi)^2}{2}\right]_\pi^{2\pi}=1\quad\left(\text{or } \frac{1}{2}+a\left[\frac{x^2}{2}-\pi x\right]_\pi^{2\pi}=1\right)$ ***A1***

$\Rightarrow a\frac{\pi^2}{2}=\frac{1}{2}$ ***A1***

$\Rightarrow a=\frac{1}{\pi^2}$ ***AG***

Note: Must obtain the exact value. Do not accept answers obtained with calculator.

METHOD 2

$0.5 +$ area of triangle $= 1$ ***R1***

area of triangle $= \frac{1}{2}\pi \times a\pi = 0.5$ ***M1A1***

Note: Award ***M1*** for correct use of area formula $= 0.5$, ***A1*** for $a\pi$.

$a = \frac{1}{\pi^2}$ ***AG***

[3 marks]

d. median is π ***A1***

[1 mark]

e. $\mu = \int_0^\pi x \cdot \frac{\sin x}{4}\mathrm{d}x + \int_\pi^{2\pi} x \cdot \frac{x-\pi}{\pi^2}\mathrm{d}x$ ***(M1)(A1)***

$= 3.40339\ldots = 3.40 \quad \left(\text{or } \frac{\pi}{4} + \frac{5\pi}{6} = \frac{13}{12}\pi\right)$ ***A1***

[3 marks]

f. For $\mu = 3.40339\ldots$

EITHER

$\sigma^2 = \int_0^\pi x^2 \cdot \frac{\sin x}{4}\mathrm{d}x + \int_\pi^{2\pi} x^2 \cdot \frac{x-\pi}{\pi^2}\mathrm{d}x - \mu^2$ ***(M1)(A1)***

OR

$\sigma^2 = \int_0^\pi (x-\mu)^2 \cdot \frac{\sin x}{4}\mathrm{d}x + \int_\pi^{2\pi} (x-\mu)^2 \cdot \frac{x-\pi}{\pi^2}\mathrm{d}x$ ***(M1)(A1)***

THEN

$= 3.866277\ldots = 3.87$ ***A1***

[3 marks]

g. $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{4}\mathrm{d}x + \int_\pi^{\frac{3\pi}{2}} \frac{x-\pi}{\pi^2}\mathrm{d}x = 0.375 \quad \left(\text{or } \frac{1}{4} + \frac{1}{8} = \frac{3}{8}\right)$ ***(M1)A1***

[2 marks]

h. $\mathrm{P}\left(\pi \le X \le 2\pi \,\middle|\, \frac{\pi}{2} \le X \le \frac{3\pi}{2}\right) = \frac{\mathrm{P}\left(\pi \le X \le \frac{3\pi}{2}\right)}{\mathrm{P}\left(\frac{\pi}{2} \le X \le \frac{3\pi}{2}\right)}$ ***(M1)(A1)***

$= \frac{\int_\pi^{\frac{3\pi}{2}} \frac{(x-\pi)}{\pi^2}\mathrm{d}x}{0.375} = \frac{0.125}{0.375} \quad \left(\text{or } = \frac{\frac{1}{8}}{\frac{3}{8}} \text{ from diagram areas}\right)$ ***(M1)***

$= \frac{1}{3} \quad (0.333)$ ***A1***

[4 marks]

Total [20 marks]

Examiners report

a. Most candidates sketched the graph correctly. In a few cases candidates did not seem familiar with the shape of the graphs and ignored the fact that the graph represented a pdf. The correct sketch assisted greatly in the rest of the question.

b. Most candidates answered this question correctly.

c. A few good proofs were seen but also many poor answers where the candidates assumed what you were trying to prove and verified numerically the result.

d. Most candidates stated the value correctly but many others showed no understanding of the concept.

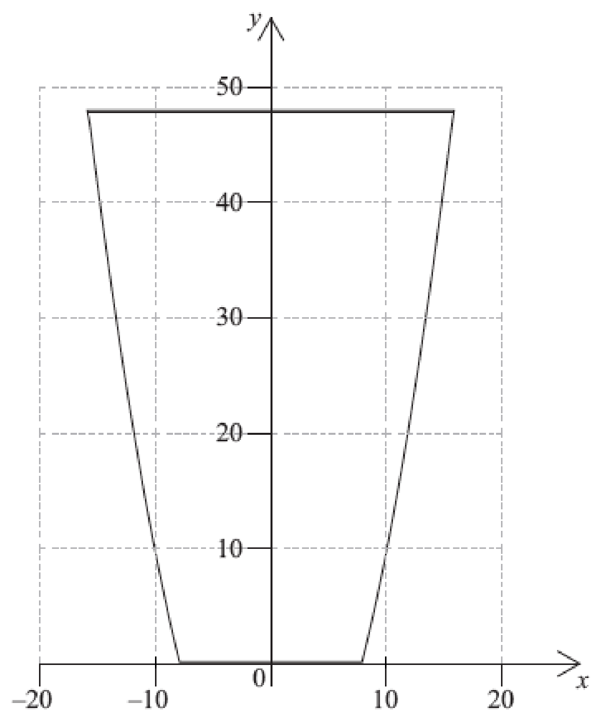
e. Many candidates scored full marks in this question; many others could not apply the formula due to difficulties in dealing with the piecewise function. For example, a number of candidates divided the final answer by two.

f. Many misconceptions were identified: use of incorrect formula (e.g. formula for discrete distributions), use of both expressions as integrand and division of the result by 2 at the end.

g. This part was fairly well done with many candidates achieving full marks.

h. Many candidates had difficulties with this part showing that the concept of conditional probability was poorly understood. The best candidates did it correctly from the sketch.

The vertical cross-section of a container is shown in the following diagram.



The curved sides of the cross-section are given by the equation $y = 0.25x^2 - 16$. The horizontal cross-sections are circular. The depth of the container is 48 cm.

a. If the container is filled with water to a depth of h cm, show that the volume, V cm^3, of the water is given by $V = 4\pi\left(\frac{h^2}{2} + 16h\right)$. [3]

b. The container, initially full of water, begins leaking from a small hole at a rate given by \(\frac{{{\text{d}}V}}{{{\text{d}}t}} = - \frac{{250\sqrt h }}{{\pi(h + 16)}}\) where t is measured in seconds. [10]

(i) Show that \(\frac{{{\text{d}}h}}{{{\text{d}}t}} = - \frac{{250\sqrt h }}{{4{\pi ^2}{{(h + 16)}^2}}}\).

(ii) State $\frac{\mathrm{d}t}{\mathrm{d}h}$ and hence show that $t = \frac{-4\pi^2}{250}\int\left(h^{\frac{3}{2}} + 32h^{\frac{1}{2}} + 256h^{-\frac{1}{2}}\right)\mathrm{d}h$.

(iii) Find, correct to the nearest minute, the time taken for the container to become empty. (60 seconds = 1 minute)

c. Once empty, water is pumped back into the container at a rate of $8.5\ \mathrm{cm^3 s^{-1}}$. At the same time, water continues leaking from the container at a rate of $\frac{250\sqrt{h}}{\pi(h+16)}\mathrm{cm^3 s^{-1}}$. [3]

Using an appropriate sketch graph, determine the depth at which the water ultimately stabilizes in the container.

Markscheme

a. attempting to use $V = \pi\int_a^b x^2 \mathrm{d}y$ ***(M1)***

attempting to express x^2 in terms of y *ie* $x^2 = 4(y+16)$ ***(M1)***

for $y = h,\ V = 4\pi\int_0^h y + 16\mathrm{d}y$ ***A1***

$V = 4\pi\left(\frac{h^2}{2} + 16h\right)$ ***AG***

[3 marks]

b. (i) **METHOD 1**

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ ***(M1)***

$\frac{\mathrm{d}V}{\mathrm{d}h} = 4\pi(h+16)$ ***(A1)***

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{4\pi(h+16)}\times\frac{-250\sqrt{h}}{\pi(h+16)}$ ***M1A1***

Note: Award ***M1*** for substitution into $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$.

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{250\sqrt{h}}{4\pi^2(h+16)^2}$ ***AG***

METHOD 2

$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi(h+16)\frac{\mathrm{d}h}{\mathrm{d}t}$ (implicit differentiation)***(M1)***

$\frac{-250\sqrt{h}}{\pi(h+16)} = 4\pi(h+16)\frac{\mathrm{d}h}{\mathrm{d}t}$ (or equivalent) ***A1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{4\pi(h+16)}\times\frac{-250\sqrt{h}}{\pi(h+16)}$ ***M1A1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{250\sqrt{h}}{4\pi^2(h+16)^2}$ ***AG***

(ii) $\frac{\mathrm{d}t}{\mathrm{d}h} = -\frac{4\pi^2(h+16)^2}{250\sqrt{h}}$ ***A1***

$t = \int -\frac{4\pi^2(h+16)^2}{250\sqrt{h}}\mathrm{d}h$ ***(M1)***

$t = \int -\frac{4\pi^2(h^2+32h+256)}{250\sqrt{h}}\mathrm{d}h$ ***A1***

$t = \frac{-4\pi^2}{250}\int\left(h^{\frac{3}{2}} + 32h^{\frac{1}{2}} + 256h^{-\frac{1}{2}}\right)\mathrm{d}h$ ***AG***

(iii) **METHOD 1**

$t = \frac{-4\pi^2}{250}\int_{48}^{0}\left(h^{\frac{3}{2}} + 32h^{\frac{1}{2}} + 256h^{-\frac{1}{2}}\right)\mathrm{d}h$ ***(M1)***

$t = 2688.756\ldots\ (\mathrm{s})$ ***(A1)***

45 minutes (correct to the nearest minute) ***A1***

METHOD 2

$t = \frac{-4\pi^2}{250}\left(\frac{2}{5}h^{\frac{5}{2}} + \frac{64}{3}h^{\frac{3}{2}} + 512h^{\frac{1}{2}}\right) + c$

when $t = 0,\ h = 48 \Rightarrow c = 2688.756\ldots \left(c = \frac{4\pi^2}{250}\left(\frac{2}{5} \times 48^{\frac{5}{2}} + \frac{64}{3} \times 48^{\frac{3}{2}} + 512 \times 48^{\frac{1}{2}}\right)\right)$ ***(M1)***

when $h = 0,\ t = 2688.756\ldots \left(t = \frac{4\pi^2}{250}\left(\frac{2}{5} \times 48^{\frac{5}{2}} + \frac{64}{3} \times 48^{\frac{3}{2}} + 512 \times 48^{\frac{1}{2}}\right)\right)$ (s) ***(A1)***

45 minutes (correct to the nearest minute) ***A1***

[10 marks]

c. **EITHER**

the depth stabilizes when $\frac{\mathrm{d}V}{\mathrm{d}t} = 0 \quad ie \quad 8.5 - \frac{250\sqrt{h}}{\pi(h+16)} = 0$ ***R1***

attempting to solve $8.5 - \frac{250\sqrt{h}}{\pi(h+16)} = 0 \quad \text{for } h$ ***(M1)***

OR

the depth stabilizes when $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \quad ie \quad \frac{1}{4\pi(h+16)}\left(8.5 - \frac{250\sqrt{h}}{\pi(h+16)}\right) = 0$ ***R1***

attempting to solve $\frac{1}{4\pi(h+16)}\left(8.5 - \frac{250\sqrt{h}}{\pi(h+16)}\right) = 0 \quad \text{for } h$ ***(M1)***

THEN

$h = 5.06\ (\text{cm})$ ***A1***

[3 marks]

Total [16 marks]

Examiners report

a. This question was done reasonably well by a large proportion of candidates. Many candidates however were unable to show the required result in part (a). A number of candidates seemingly did not realize how the container was formed while other candidates attempted to fudge the result.

b. Part (b) was quite well done. In part (b) (i), most candidates were able to correctly calculate $\frac{\mathrm{d}V}{\mathrm{d}h}$ and correctly apply a related rates expression to show the given result. Some candidates however made a sign error when stating $\frac{\mathrm{d}V}{\mathrm{d}t}$. A large number of candidates successfully answered part (b) (ii). In part (b) (iii), successful candidates either set up and calculated an appropriate definite integral or antidifferentiated and found that $t = C$ when $h = 0$.

c. In part (c), a pleasing number of candidates realized that the water depth stabilized when either $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, sketched an appropriate graph and found the correct value of h. Some candidates misinterpreted the situation and attempted to find the coordinates of the local minimum of their graph.

The seventh, third and first terms of an arithmetic sequence form the first three terms of a geometric sequence.

The arithmetic sequence has first term a and non-zero common difference d.

a. Show that $d = \frac{a}{2}$. [3]

b. The seventh term of the arithmetic sequence is 3. The sum of the first n terms in the arithmetic sequence exceeds the sum of the first n terms in the geometric sequence by at least 200. [6]

Find the least value of n for which this occurs.

Markscheme

a. using $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$ to form $\frac{a+2d}{a+6d} = \frac{a}{a+2d}$ ***(M1)***

$a(a+6d) = (a+2d)^2$ ***A1***

$2d(2d-a) = 0$ (or equivalent) ***A1***

since $d \neq 0 \Rightarrow d = \frac{a}{2}$ ***AG***

[3 marks]

b. substituting $d = \frac{a}{2}$ into $a + 6d = 3$ and solving for a and d ***(M1)***

$a = \frac{3}{4}$ and $d = \frac{3}{8}$ ***(A1)***

$r = \frac{1}{2}$ ***A1***

$\frac{n}{2}\left(2 \times \frac{3}{4} + (n-1)\frac{3}{8}\right) - \frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} \geq 200$ ***(A1)***

attempting to solve for n ***(M1)***

$n \geq 31.68\ldots$

so the least value of n is 32 ***A1***

[6 marks]

Total [9 marks]

Examiners report

a. Part (a) was reasonably well done. A number of candidates used $r = \frac{u_1}{u_2} = \frac{u_2}{u_3}$ rather than $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$. This invariably led to candidates obtaining $r = 2$ in part (b).

b. In part (b), most candidates were able to correctly find the first term and the common difference for the arithmetic sequence. However a number of candidates either obtained $r = 2$ via means described in part (a) or confused the two sequences and used $u_1 = \frac{3}{4}$ for the geometric sequence.

Let $z = r(\cos\alpha + \mathrm{i}\sin\alpha)$, where α is measured in degrees, be the solution of $z^5 - 1 = 0$ which has the smallest positive argument.

a. (i) Use the binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$. [6]

(ii) Hence use De Moivre's theorem to prove

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta.$$

(iii) State a similar expression for $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$.

b. Find the value of r and the value of α. [4]

c. Using (a) (ii) and your answer from (b) show that $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$. [4]

d. Hence express $\sin 72^\circ$ in the form $\frac{\sqrt{a+b\sqrt{c}}}{d}$ where $a,\ b,\ c,\ d \in \mathbb{Z}$. [5]

Markscheme

a. (i) $(\cos\theta + \mathrm{i}\sin\theta)^5$

$= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta + 10\mathrm{i}^2\cos^3\theta\sin^2\theta +$

$10\mathrm{i}^3\cos^2\theta\sin^3\theta + 5\mathrm{i}^4\cos\theta\sin^4\theta + \mathrm{i}^5\sin^5\theta$ ***A1A1***

$(= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta -$

$10\mathrm{i}\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + \mathrm{i}\sin^5\theta)$

Note: Award first ***A1*** for correct binomial coefficients.

(ii) $(\mathrm{cis}\theta)^5 = \mathrm{cis}5\theta = \cos 5\theta + \mathrm{i}\sin 5\theta$ ***M1***

$= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10\mathrm{i}\cos^2\theta\sin^3\theta +$

$5\cos\theta\sin^4\theta + \mathrm{i}\sin^5\theta$ ***A1***

Note: Previous line may be seen in (i)

equating imaginary terms ***M1***

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ ***AG***

(iii) equating real terms

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ ***A1***

[6 marks]

b. $(r\mathrm{cis}\alpha)^5 = 1 \Rightarrow r^5\mathrm{cis}5\alpha = 1\mathrm{cis}0$ ***M1***

$r^5 = 1 \Rightarrow r = 1$ ***A1***

$5\alpha = 0 \pm 360k,\ k \in \mathbb{Z} \Rightarrow a = 72k$ ***(M1)***

$\alpha = 72^\circ$ ***A1***

Note: Award ***M1A0*** if final answer is given in radians.

[4 marks]

c. use of $\sin(5 \times 72) = 0$ **OR** the imaginary part of 1 is 0 ***(M1)***

$0 = 5\cos^4\alpha\sin\alpha - 10\cos^2\alpha\sin^3\alpha + \sin^5\alpha$ ***A1***

$\sin\alpha \neq 0 \Rightarrow 0 = 5(1 - \sin^2\alpha)^2 - 10(1 - \sin^2\alpha)\sin^2\alpha + \sin^4\alpha$ ***M1***

Note: Award ***M1*** for replacing $\cos^2\alpha$.

$0 = 5(1 - 2\sin^2\alpha + \sin^4\alpha) - 10\sin^2\alpha + 10\sin^4\alpha + \sin^4\alpha$ ***A1***

Note: Award ***A1*** for any correct simplification.

so $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$ ***AG***

[4 marks]

d. $\sin^2\alpha = \frac{20\pm\sqrt{400-320}}{32}$ ***M1A1***

$\sin\alpha = \pm\sqrt{\frac{20\pm\sqrt{80}}{32}}$

$\sin\alpha = \frac{\pm\sqrt{10\pm2\sqrt{5}}}{4}$ ***A1***

Note: Award ***A1*** regardless of signs. Accept equivalent forms with integral denominator, simplification may be seen later.

as $72 > 60,\ \sin 72 > \frac{\sqrt{3}}{2} = 0.866\ldots$ we have to take both positive signs (or equivalent argument) ***R1***

Note: Allow verification of correct signs with calculator if clearly stated

$\sin 72 = \frac{\sqrt{10+2\sqrt{5}}}{4}$ ***A1***

[5 marks]

Total [19 marks]

Examiners report

a. In part (i) many candidates tried to multiply it out the binomials rather than using the binomial theorem. In parts (ii) and (iii) many candidates showed poor understanding of complex numbers and made no attempt to equate real and imaginary parts. In a some cases the correct answer to part (iii) was seen although it was unclear how it was obtained.

b. This question was poorly done. Very few candidates made a good attempt to apply De Moivre's theorem and most of them could not even equate the moduli to obtain r.

c. This question was poorly done. From the few candidates that attempted it, many candidates started by writing down what they were trying to prove and made no progress.

d. Very few made a serious attempt to answer this question. Also very few realised that they could use the answers given in part (c) to attempt this part.

Consider $f(x) = -1 + \ln\left(\sqrt{x^2-1}\right)$

The function f is defined by $f(x) = -1 + \ln\left(\sqrt{x^2-1}\right),\ x \in D$

The function g is defined by $g(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right),\ x \in]1,\ \infty[$.

a. Find the largest possible domain D for f to be a function. [2]

b. Sketch the graph of $y = f(x)$ showing clearly the equations of asymptotes and the coordinates of any intercepts with the axes. [3]

c. Explain why f is an even function. [1]

d. Explain why the inverse function f^{-1} does not exist. [1]

e. Find the inverse function g^{-1} and state its domain. [4]

f. Find $g'(x)$. [3]

g.i. Hence, show that there are no solutions to $g'(x) = 0$; [2]

g.ii. Hence, show that there are no solutions to $(g^{-1})'(x) = 0$. [2]

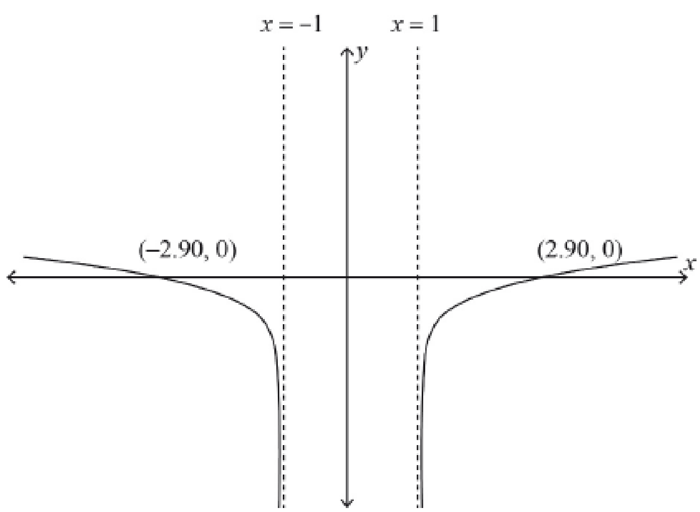
Markscheme

a. $x^2 - 1 > 0$ ***(M1)***

$x < -1$ or $x > 1$ ***A1***

[2 marks]

b.



shape ***A1***

$x = 1$ and $x = -1$ ***A1***

x-intercepts ***A1***

[3 marks]

c. **EITHER**

f is symmetrical about the y-axis ***R1***

OR

$f(-x) = f(x)$ ***R1***

[1 mark]

d. **EITHER**

f is not one-to-one function ***R1***

OR

horizontal line cuts twice ***R1***

Note: Accept any equivalent correct statement.

[1 mark]

e. $x = -1 + \ln\left(\sqrt{y^2 - 1}\right)$ ***M1***

$e^{2x+2} = y^2 - 1$ ***M1***

$g^{-1}(x) = \sqrt{e^{2x+2} + 1},\ x \in \mathbb{R}$ ***A1A1***

[4 marks]

f. $g'(x) = \frac{1}{\sqrt{x^2-1}} \times \frac{2x}{2\sqrt{x^2-1}}$ ***M1A1***

$g'(x) = \frac{x}{x^2-1}$ ***A1***

[3 marks]

g.i. $g'(x) = \frac{x}{x^2-1} = 0 \Rightarrow x = 0$ ***M1***

which is not in the domain of g (hence no solutions to $g'(x) = 0$) ***R1***

[2 marks]

g.ii. $(g^{-1})'(x) = \frac{e^{2x+2}}{\sqrt{e^{2x+2}+1}}$ ***M1***

as $e^{2x+2} > 0 \Rightarrow (g^{-1})'(x) > 0$ so no solutions to $(g^{-1})'(x) = 0$ ***R1***

Note: Accept: equation $e^{2x+2} = 0$ has no solutions.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g.i. [N/A]
g.ii. [N/A]

Let $f(x) = x^4 + 0.2x^3 - 5.8x^2 - x + 4,\ x \in \mathbb{R}$.

The domain of f is now restricted to $[0,\ a]$.

Let $g(x) = 2\sin(x-1) - 3,\ \ -\frac{\pi}{2} + 1 \leqslant x \leqslant \frac{\pi}{2} + 1.$

a. Find the solutions of $f(x) > 0$. [3]

b. For the curve $y = f(x)$. [5]

(i) Find the coordinates of both local minimum points.

(ii) Find the x-coordinates of the points of inflexion.

c.i. Write down the largest value of a for which f has an inverse. Give your answer correct to 3 significant figures. [2]

c.ii. For this value of *a* sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing clearly the coordinates of the end points of each curve. [2]

c.iii. Solve $f^{-1}(x) = 1$. [2]

d.i. Find an expression for $g^{-1}(x)$, stating the domain. [4]

d.ii. Solve $(f^{-1} \circ g)(x) < 1$. [4]

Markscheme

a. valid method *eg*, sketch of curve or critical values found ***(M1)***

$x < -2.24,\ x > 2.24,$ ***A1***

$-1 < x < 0.8$ ***A1***

Note: Award ***M1A1A0*** for correct intervals but with inclusive inequalities.

[3 marks]

b. (i) $(1.67,\ -5.14),\ (-1.74,\ -3.71)$ ***A1A1***

Note: Award ***A1A0*** for any two correct terms.

(ii) $f'(x) = 4x^3 + 0.6x^2 - 11.6x - 1$

$f''(x) = 12x^2 + 1.2x - 11.6 = 0$ ***(M1)***

$-1.03,\ 0.934$ ***A1A1***

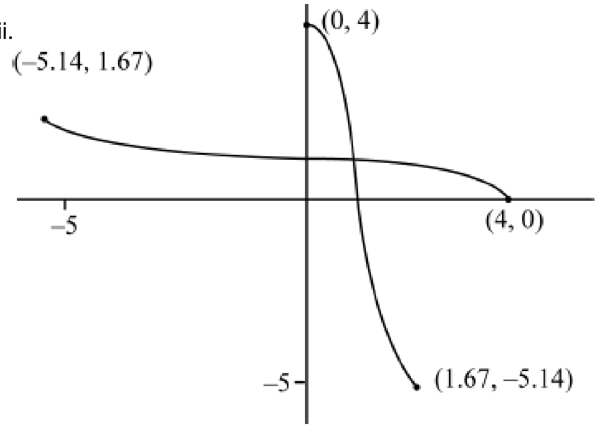
Note: ***M1*** should be awarded if graphical method to find zeros of $f''(x)$ or turning points of $f'(x)$ is shown.

[5 marks]

c.i. 1.67 ***A1***

[2 marks]

c.ii.



M1A1A1

Note: Award ***M1*** for reflection of their $y = f(x)$ in the line $y = x$ provided their f is one-one.

A1 for $(0,\ 4)$, $(4,\ 0)$ (Accept axis intercept values) ***A1*** for the other two sets of coordinates of other end points

[2 marks]

c.iii. $x = f(1)$ ***M1***

$= -1.6$ ***A1***

[2 marks]

d.i. $y = 2\sin(x-1) - 3$

$x = 2\sin(y-1) - 3$ ***(M1)***

$\left(g^{-1}(x) =\right)\ \arcsin\left(\frac{x+3}{2}\right) + 1$ ***A1***

$-5 \leqslant x \leqslant -1$ ***A1A1***

Note: Award ***A1*** for –5 and –1, and ***A1*** for correct inequalities if numbers are reasonable.

[8 marks]

d.ii. $f^{-1}\left(g(x)\right) < 1$

$g(x) > -1.6$ ***(M1)***

$x > g^{-1}(-1.6) = 1.78$ ***(A1)***

Note: Accept = in the above.

$1.78 < x \leqslant \frac{\pi}{2} + 1$ ***A1A1***

Note: ***A1*** for $x > 1.78$ (allow ≥) and ***A1*** for $x \leqslant \frac{\pi}{2} + 1$.

[4 marks]

Examiners report

a. Parts (a) and (b) were well answered, with considerably less success in part (c). Surprisingly few students were able to reflect the curve in $y = x$ satisfactorily, and many were not making their sketch using the correct domain.

b. Parts (a) and (b) were well answered, with considerably less success in part (c). Surprisingly few students were able to reflect the curve in $y = x$ satisfactorily, and many were not making their sketch using the correct domain.

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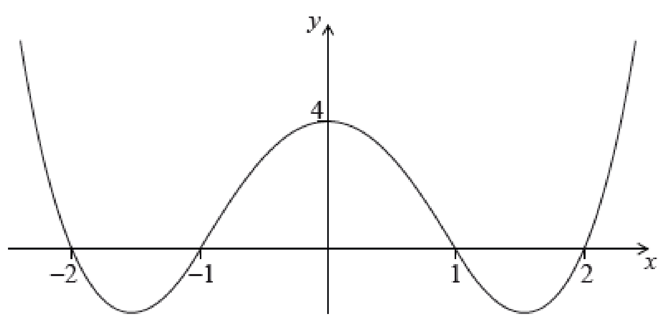
c.iii. Parts (a) and (b) were well answered, with considerably less success in part (c). Surprisingly few students were able to reflect the curve in $y = x$ satisfactorily, and many were not making their sketch using the correct domain.

d.i. Part d(i) was generally well done, but there were few correct answers for d(ii).

d.ii. Part d(i) was generally well done, but there were few correct answers for d(ii).

Let $f(x) = |x| - 1$.

(a) The graph of $y = g(x)$ is drawn below.



(i) Find the value of $(f \circ g)(1)$.

(ii) Find the value of $(f \circ g \circ g)(1)$.

(iii) Sketch the graph of $y = (f \circ g)(x)$.

(b) (i) Sketch the graph of $y = f(x)$.

(ii) State the zeros of f.

(c) (i) Sketch the graph of $y = (f \circ f)(x)$.

(ii) State the zeros of $f \circ f$.

(d) Given that we can denote $\underbrace{f \circ f \circ f \circ \ldots \circ f}_{n \text{ times}}$ as f^n,

(i) find the zeros of f^3;

(ii) find the zeros of f^4;

(iii) deduce the zeros of f^8.

(e) The zeros of f^{2n} are $a_1,\ a_2,\ a_3,\ \ldots,\ a_N$.

(i) State the relation between n and N;

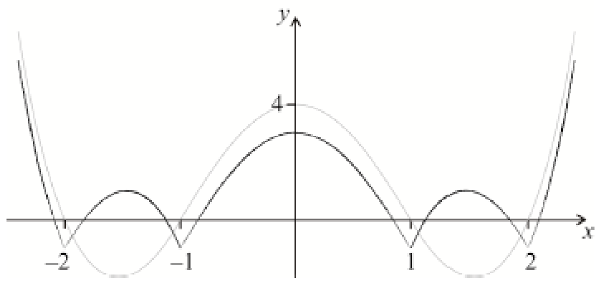
(ii) Find, and simplify, an expression for $\sum_{r=1}^{N} |a_r|$ in terms of n.

Markscheme

(a) (i) $f(0) = -1$ ***(M1)A1***

(ii) $(f \circ g)(0) = f(4) = 3$ ***A1***

(iii)

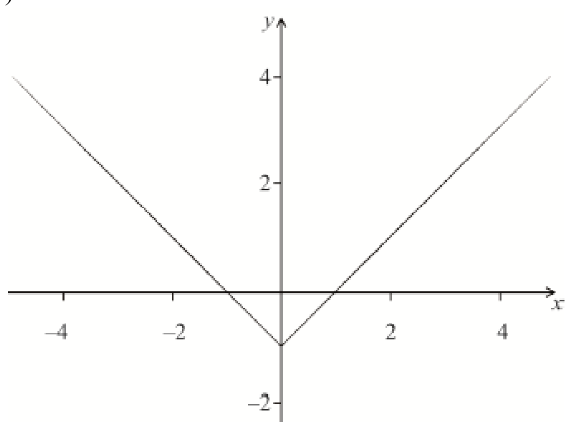


(M1)A1

Note: Award ***M1*** for evidence that the lower part of the graph has been reflected and ***A1*** correct shape with *y*-intercept below 4.

[5 marks]

(b) (i)



(M1)A1

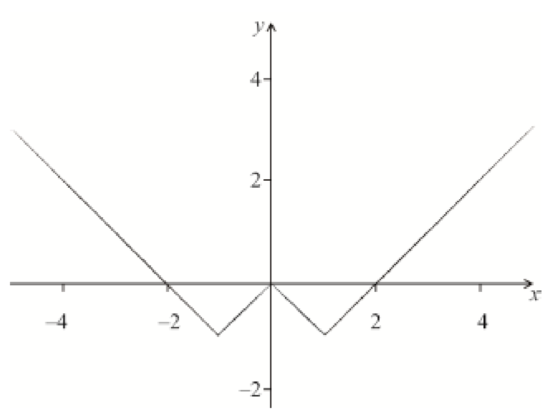
Note: Award ***M1*** for any translation of $y = |x|$.

(ii) ± 1 ***A1***

Note: Do not award the ***A1*** if coordinates given, but do not penalise in the rest of the question

[3 marks]

(c) (i)



(M1)A1

Note: Award ***M1*** for evidence that lower part of (b) has been reflected in the *x*-axis and translated.

(ii) $0,\ \pm 2$ ***A1***

[3 marks]

(d) (i) $\pm 1,\ \pm 3$ ***A1***

(ii) $0,\ \pm 2,\ \pm 4$ ***A1***

(iii) $0,\ \pm 2,\ \pm 4,\ \pm 6,\ \pm 8$ ***A1***

[3 marks]

(e) (i) $(1,3), (2,5),\ \ldots$ ***(M1)***

$N = 2n + 1$ ***A1***

(ii) Using the formula of the sum of an arithmetic series ***(M1)***

EITHER

$4(1 + 2 + 3 + \ldots + n) = \frac{4}{2}n(n+1)$

$= 2n(n+1)$ ***A1***

OR

$2(2 + 4 + 6 + \ldots + 2n) = \frac{2}{2}n(2n+2)$

$= 2n(n+1)$ ***A1***

[4 marks]

Total [18 marks]

Examiners report

[N/A]

The function f is given by $f(x) = \frac{3x^2+10}{x^2-4},\ x \in \mathbb{R},\ x \neq 2,\ x \neq -2.$

a. Prove that f is an even function. [2]

b.i. Sketch the graph $y = f(x)$. [3]

b.ii. Write down the range of f. [2]

Markscheme

a. $f(-x) = \frac{3(-x)^2+10}{(-x)^2-4}$ ***A1***

$= \frac{3x^2+10}{x^2-4} = f(x)$

$f(x) = f(-x)$ ***R1***

hence this is an even function ***AG***

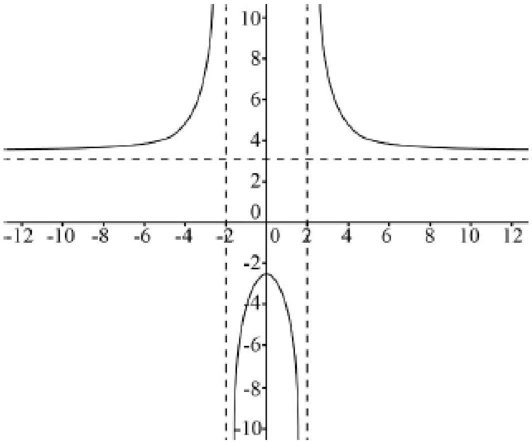
Note: Award ***A1R1*** for the statement, all the powers are even hence $f(x) = f(-x)$.

Note: Just stating all the powers are even is ***A0R0***.

Note: Do not accept arguments based on the symmetry of the graph.

[2 marks]

b.i.



correct shape in 3 parts which are asymptotic and symmetrical ***A1***

correct vertical asymptotes clear at 2 and –2 ***A1***

correct horizontal asymptote clear at 3 ***A1***

[3 marks]

b.ii. $f(x) > 3$ ***A1***

$f(x) \leqslant -2.5$ ***A1***

[2 marks]

Examiners report

a. Most candidates were able to prove that a function was even, although many attempted to show special cases, rather than a general proof. Many lost marks through not showing the asymptotes on their sketch. Marks were commonly lost in incorrect use of inequalities for the range of the function.

b.i. Most candidates were able to prove that a function was even, although many attempted to show special cases, rather than a general proof. Many lost marks through not showing the asymptotes on their sketch. Marks were commonly lost in incorrect use of inequalities for the range of the function.

b.ii. Most candidates were able to prove that a function was even, although many attempted to show special cases, rather than a general proof. Many lost marks through not showing the asymptotes on their sketch. Marks were commonly lost in incorrect use of inequalities for the range of the function.

Find the set of values of x for which $\left|0.1x^2 - 2x + 3\right| < \log_{10}x$.

Markscheme

Attempting to solve $\left|0.1x^2 - 2x + 3\right| = \log_{10}x$ numerically or graphically. ***(M1)***

$x = 1.52,\ 1.79$ ***(A1)(A1)***

$x = 17.6,\ 19.1$ ***(A1)***

$(1.52 < x < 1.79) \cup (17.6 < x < 19.1)$ ***A1A1*** ***N2***

[6 marks]

Examiners report

This question was generally not well done. A number of candidates attempted an ‘ill-fated’ algebraic approach. Most candidates who used their GDC were able to correctly locate one inequality. The few successful candidates were able to employ a suitable window or suitable window(s) to correctly locate both inequalities.

Consider the functions $f(x) = x^3 + 1$ and $g(x) = \frac{1}{x^3+1}$. The graphs of $y = f(x)$ and $y = g(x)$ meet at the point (0, 1) and one other point, P.

a. Find the coordinates of P. [1]

b. Calculate the size of the acute angle between the tangents to the two graphs at the point P. [4]

Markscheme

a. $x^3 + 1 = \frac{1}{x^3+1}$

$(-1.26, -1)$ $\left(= \left(-\sqrt[3]{2}, -1\right)\right)$ ***A1***

[1 mark]

b. $f'(-1.259...) = 4.762\ldots$ $(3 \times 2^{\frac{2}{3}})$ ***A1***

$g'(-1.259...) = -4.762\ldots$ $(-3 \times 2^{\frac{2}{3}})$ ***A1***

required angle $= 2\arctan\left(\frac{1}{4.762...}\right)$ ***M1***

$= 0.414$ (accept 23.7) ***A1***

Note: Accept alternative methods including finding the obtuse angle first.

[4 marks]

Examiners report

a. In part (a) almost all candidates obtained the correct answer, either in numerical form or in exact form. Although many candidates scored one mark in (b), for one gradient, few scored any more. Successful candidates almost always adopted a vector approach to finding the angle between the two tangents, rather than using trigonometry.

b. In part (a) almost all candidates obtained the correct answer, either in numerical form or in exact form. Although many candidates scored one mark in (b), for one gradient, few scored any more. Successful candidates almost always adopted a vector approach to finding the angle between the two tangents, rather than using trigonometry.

(a) Simplify the difference of binomial coefficients

$$\binom{n}{3} - \binom{2n}{2}, \text{ where } n \geqslant 3.$$

(b) Hence, solve the inequality

$$\binom{n}{3} - \binom{2n}{2} > 32n, \text{ where } n \geqslant 3.$$

Markscheme

(a) the expression is

$\frac{n!}{(n-3)!3!} - \frac{(2n)!}{(2n-2)!2!}$ ***(A1)***

$\frac{n(n-1)(n-2)}{6} - \frac{2n(2n-1)}{2}$ ***M1A1***

$= \frac{n(n^2-15n+8)}{6} \left(= \frac{n^3-15n^2+8n}{6}\right)$ ***A1***

(b) the inequality is

$\frac{n^3-15n^2+8n}{6} > 32n$

attempt to solve cubic inequality or equation ***(M1)***

$n^3 - 15n^2 - 184n > 0 \quad n(n-23)(n+8) > 0$

$n > 23 \quad (n \geqslant 24)$ ***A1***

[6 marks]

Examiners report

Part(a) - Although most understood the notation, few knew how to simplify the binomial coefficients.

Part(b) - Many were able to solve the cubic, but some failed to report their answer as an integer inequality.

The function $f(x) = 4x^3 + 2ax - 7a$, $a \in \mathbb{R}$, leaves a remainder of -10 when divided by $(x - a)$.

a. Find the value of a . [3]

b. Show that for this value of a there is a unique real solution to the equation $f(x) = 0$. [2]

Markscheme

a. $f(a) = 4a^3 + 2a^2 - 7a = -10$ ***M1***

$4a^3 + 2a^2 - 7a + 10 = 0$

$(a+2)\left(4a^2 - 6a + 5\right) = 0$ or sketch or GDC ***(M1)***

$a = -2$ ***A1***

[3 marks]

b. substituting $a = -2$ into $f(x)$

$f(x) = 4x^3 - 4x + 14 = 0$ ***A1***

EITHER

graph showing unique solution which is indicated (must include max and min) ***R1***

OR

convincing argument that only one of the solutions is real ***R1***

(−1.74, 0.868 ±1.12i)

[5 marks]

Examiners report

a. Candidates found this question surprisingly challenging. The most straightforward approach was use of the Remainder Theorem but a significant number of candidates seemed unaware of this technique. This lack of knowledge led many candidates to attempt an algebraically laborious use of long division. In (b) a number of candidates did not seem to appreciate the significance of the word unique and hence found it difficult to provide sufficient detail to make a meaningful argument. However, most candidates did recognize that they needed a technological approach when attempting (b).

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---

a. Write down the quadratic expression $2x^2 + x - 3$ as the product of two linear factors. [1]

b. Hence, or otherwise, find the coefficient of x in the expansion of $\left(2x^2 + x - 3\right)^8$. [4]

Markscheme

a. $2x^2 + x - 3 = (2x + 3)(x - 1)$ ***A1***

Note: Accept $2\left(x + \frac{3}{2}\right)(x - 1)$.

Note: Either of these may be seen in (b) and if so ***A1*** should be awarded.

[1 mark]

b. **EITHER**

$\left(2x^2+x-3\right)^8=(2x+3)^8(x-1)^8$ ***M1***

$=\left(3^8+8\left(3^7\right)(2x)+\ldots\right)\left((-1)^8+8(-1)^7(x)+\ldots\right)$ ***(A1)***

coefficient of $x=3^8\times 8\times(-1)^7+3^7\times 8\times 2\times(-1)^8$ ***M1***

$=-17\,496$ ***A1***

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

OR

$\left(2x^2+x-3\right)^8=\left(3-\left(x-2x^2\right)\right)^8$ ***M1***

$=3^8+8\left(-\left(x-2x^2\right)\left(3^7\right)+\ldots\right)$ ***(A1)***

coefficient of $x=8\times(-1)\times 3^7$ ***M1***

$=-17\,496$ ***A1***

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

[4 marks]

Examiners report

a. Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8^{th} power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.

b. Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8^{th} power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.

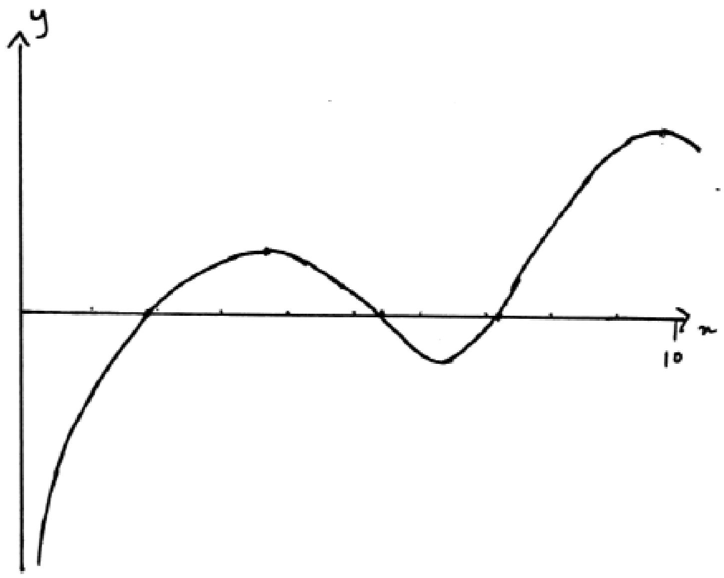
Consider $f(x)=\ln x-\mathrm{e}^{\cos x},\ 0<x\leqslant 10$.

a. Sketch the graph of $y=f(x)$, stating the coordinates of any maximum and minimum points and points of intersection with the x-axis. [5]

b. Solve the inequality $\ln x\leqslant \mathrm{e}^{\cos x},\ 0<x\leqslant 10$. [2]

Markscheme

a.



A correct graph shape for $0 < x \leqslant 10$ ***A1***
maxima (3.78, 0.882) and (9.70, 1.89) ***A1***
minimum (6.22, –0.885) ***A1***
x-axis intercepts (1.97, 0), (5.24, 0) and (7.11, 0) ***A2***

Note: Award ***A1*** if two *x*-axis intercepts are correct.

[5 marks]

b. $0 < x \leqslant 1.97$ ***A1***

$5.24 \leqslant x \leqslant 7.11$ ***A1***
[2 marks]

Examiners report

a. Part (a) was reasonably well done although more care was required when showing correct endpoint behaviour. A number of sketch graphs suggested the existence of either a vertical axis intercept or displayed an open circle on the vertical axis. A large number of candidates did not state the coordinates of the various key features correct to three significant figures. A large number of candidates did not locate the maximum near $x = 10$. Most candidates were able to locate the *x*-axis intercepts and the minimum. A few candidates unfortunately sketched a graph from a GDC set in degrees.

b. In part (b), a number of candidates identified the correct critical values but used incorrect inequality signs. Some candidates attempted to solve the inequality algebraically.

The function *f* is defined as $f(x) = -3 + \frac{1}{x-2}$, $x \neq 2$.

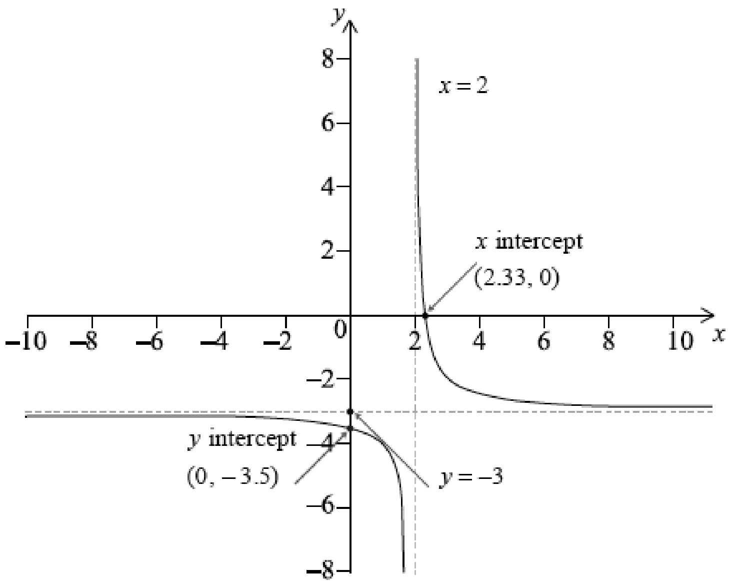
a. (i) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts. [4]

(ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts.

b. Find the inverse function f^{-1}, stating its domain. [4]

Markscheme

a.



A1A1A1

Note: Award ***A1*** for correct shape, ***A1*** for $x = 2$ clearly stated and ***A1*** for $y = -3$ clearly stated.

x intercept (2.33, 0) and y intercept (0, –3.5) ***A1***

Note: Accept –3.5 and 2.33 (7/3) marked on the correct axes.

[4 marks]

b. $x = -3 + \frac{1}{y-2}$ ***M1***

Note: Award ***M1*** for interchanging *x* and *y* (can be done at a later stage).

$x + 3 = \frac{1}{y-2}$
$y - 2 = \frac{1}{x+3}$ ***M1***

Note: Award ***M1*** for attempting to make *y* the subject.

$f^{-1}(x) = 2 + \frac{1}{x+3}\left(= \frac{2x+7}{x+3}\right),\ x \neq -3$ ***A1A1***

Note: Award ***A1*** only if $f^{-1}(x)$ is seen. Award ***A1*** for the domain.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

A Chocolate Shop advertises free gifts to customers that collect three vouchers. The vouchers are placed at random into 10% of all chocolate bars sold at this shop. Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $\mathrm{P}(X = n)$ be the probability that Kati obtains her third voucher on the $n\mathrm{th}$ bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

It is given that $\mathrm{P}(X = n) = \frac{n^2+an+b}{2000} \times 0.9^{n-3}$ for $n \geqslant 3,\ n \in \mathbb{N}$.

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

a. Show that $\mathrm{P}(X = 3) = 0.001$ and $\mathrm{P}(X = 4) = 0.0027$. [3]

b. Find the values of the constants a and b. [5]

c. Deduce that $\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)} = \frac{0.9(n-1)}{n-3}$ for $n > 3$. [4]

d. (i) Hence show that X has two modes m_1 and m_2. [5]

(ii) State the values of m_1 and m_2.

e. Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]

Markscheme

a. $\mathrm{P}(X = 3) = (0.1)^3$ ***A1***

$= 0.001$ ***AG***

$\mathrm{P}(X = 4) = \mathrm{P}(VV\bar{V}V) + \mathrm{P}(V\bar{V}VV) + \mathrm{P}(\bar{V}VVV)$ ***(M1)***

$= 3 \times (0.1)^3 \times 0.9$ (or equivalent) ***A1***

$= 0.0027$ ***AG***

[3 marks]

b. **METHOD 1**

attempting to form equations in a and b ***M1***

$\frac{9+3a+b}{2000} = \frac{1}{1000}\ (3a + b = -7)$ ***A1***

$\frac{16+4a+b}{2000} \times \frac{9}{10} = \frac{27}{10\,000}\ (4a + b = -10)$ ***A1***

attempting to solve simultaneously ***(M1)***

$a = -3,\ b = 2$ ***A1***

METHOD 2

$\mathrm{P}(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$ ***M1***

$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$ ***(M1)A1***

$= \frac{n^2-3n+2}{2000} \times 0.9^{n-3}$ ***A1***

$a = -3, b = 2$ ***A1***

Note: Condone the absence of 0.9^{n-3} in the determination of the values of a and b.

[5 marks]

c. **METHOD 1**

EITHER

$\mathrm{P}(X=n)=\frac{n^2-3n+2}{2000}\times 0.9^{n-3}$ ***(M1)***

OR

$\mathrm{P}(X=n)=\binom{n-1}{2}\times 0.1^3\times 0.9^{n-3}$ ***(M1)***

THEN

$=\frac{(n-1)(n-2)}{2000}\times 0.9^{n-3}$ ***A1***

$\mathrm{P}(X=n-1)=\frac{(n-2)(n-3)}{2000}\times 0.9^{n-4}$ ***A1***

$\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}=\frac{(n-1)(n-2)}{(n-2)(n-3)}\times 0.9$ ***A1***

$=\frac{0.9(n-1)}{n-3}$ ***AG***

METHOD 2

$\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}=\frac{\frac{n^2-3n+2}{2000}\times 0.9^{n-3}}{\frac{(n-1)^2-3(n-1)+2}{2000}\times 0.9^{n-4}}$ ***(M1)***

$=\frac{0.9(n^2-3n+2)}{(n^2-5n+6)}$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$=\frac{0.9(n-1)(n-2)}{(n-2)(n-3)}$ ***A1***

$=\frac{0.9(n-1)}{n-3}$ ***AG***

[4 marks]

d. (i) attempting to solve $\frac{0.9(n-1)}{n-3}=1$ for n ***M1***

$n=21$ ***A1***

$\frac{0.9(n-1)}{n-3}<1\Rightarrow n>21$ ***R1***

$\frac{0.9(n-1)}{n-3}>1\Rightarrow n<21$ ***R1***

X has two modes ***AG***

Note: Award ***R1R1*** for a clearly labelled graphical representation of the two inequalities (using $\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}$).

(ii) the modes are 20 and 21 ***A1***

[5 marks]

e. **METHOD 1**

$Y\sim\mathrm{B}(x,\ 0.1)$ ***(A1)***

attempting to solve $\mathrm{P}(Y \geqslant 3) > 0.5$ (or equivalent *eg* $1 - \mathrm{P}(Y \leqslant 2) > 0.5$) for x ***(M1)***

Note: Award ***(M1)*** for attempting to solve an equality (obtaining $x = 26.4$).

$x = 27$ ***A1***

METHOD 2

$\sum_{n=0}^{x} \mathrm{P}(X = n) > 0.5$ ***(A1)***

attempting to solve for x ***(M1)***

$x = 27$ ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

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a. A function f is defined by $f(x) = (x+1)(x-1)(x-5),\ x \in \mathbb{R}$. [3]

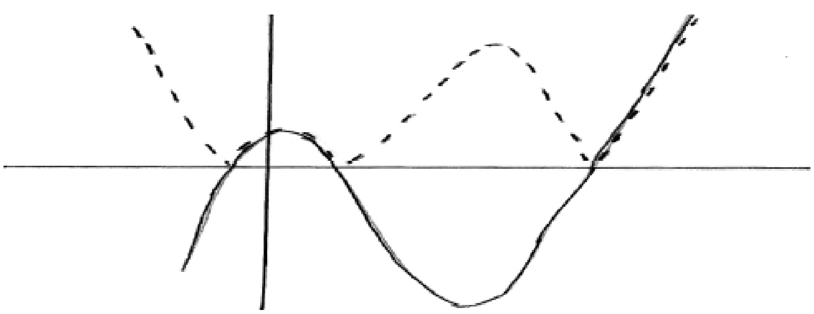
Find the values of x for which $f(x) < |f(x)|$.

b. A function g is defined by $g(x) = x^2 + x - 6,\ x \in \mathbb{R}$. [7]

Find the values of x for which $g(x) < \frac{1}{g(x)}$.

Markscheme

a.



as roots of $f(x) = 0$ are $-1,\ 1,\ 5$ ***(M1)***

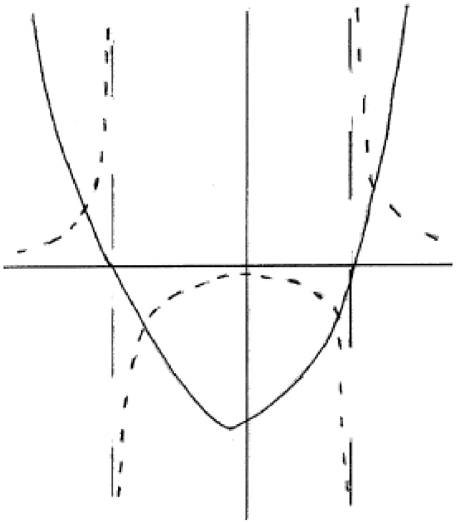
solution is $]-\infty,\ -1[\cup]1,\ 5[\quad (x < -1 \quad \text{or} \quad 1 < x < 5)$ ***A1A1***

Note: Award ***A1A0*** for closed intervals.

[3 marks]

b. **METHOD 1**

$\left(\text{graphs of } g(x) \text{ and } \frac{1}{g(x)}\right)$



roots of $g(x) = 0$ are -3 and 2 ***(M1)(A1)***

Notes: Award ***M1*** if quadratic graph is drawn or two roots obtained.

Roots may be indicated anywhere eg asymptotes on graph or in inequalities below.

the intersections of the graphs $g(x)$ and of $1/g(x)$

are $-3.19, \ -2.79, \ 1.79, \ 2.19$ ***(M1)(A1)***

Note: Award ***A1*** for at least one of the values above seen anywhere.

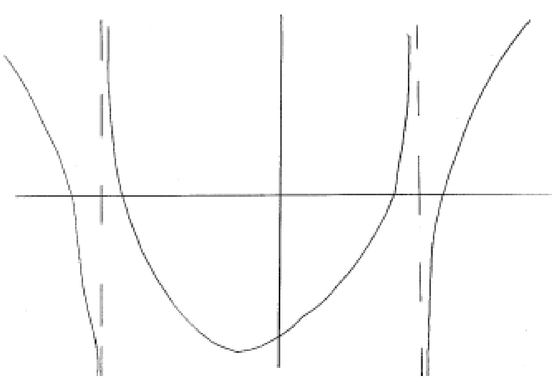
solution is $]-3.19, \ -3[\cup]-2.79, \ 1.79[\cup]2, \ 2.19[$

$(-3.19 < x < -3 \quad \text{or} \quad -2.79 < x < 1.79 \quad \text{or} \quad 2 < x < 2.19)$ ***A1A1A1***

Note: Award ***A1A1A0*** for closed intervals.

METHOD 2

$\left(\text{graph of } g(x) - \frac{1}{g(x)}\right)$



asymptotes at $x = -3$ and $x = 2$ ***(M1)(A1)***

Note: May be indicated on the graph.

roots of graph are $-3.19, \ -2.79, \ 1.79, \ 2.19$ ***(M1)(A1)***

Note: Award ***A1*** for at least one of the values above seen anywhere.

solution is (when graph is negative)

$]-3.19,\ -3[\cup]-2.79,\ 1.79[\cup]2,\ 2.19[$

$(-3.19 < x < -3 \quad \text{or} \quad -2.79 < x < 1.79 \quad \text{or} \quad 2 < x < 2.19)$ ***A1A1A1***

Note: Award ***A1A1A0*** for closed intervals.

[7 marks]

Total [10 marks]

Examiners report

a. In general part (a) was performed correctly, with the vast majority of candidates stating the correct open intervals as required.

b. In part (b) many candidates scored a few marks by just finding intersection points and equations of asymptotes; many other candidates showed difficulties in manipulating inequalities and ignored the fact that the quantities could be negative. Candidates that used the graph well managed to achieve full marks. Unfortunately many sketches were very crudely drawn hence they were of limited value for assessment purposes.

Consider the function $f(x) = \frac{\sqrt{x}}{\sin x},\ 0 < x < \pi$.

Consider the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = \frac{\pi}{6},\ x = \frac{\pi}{3}$.

a.i. Show that the x-coordinate of the minimum point on the curve $y = f(x)$ satisfies the equation $\tan x = 2x$. [5]

a.ii. Determine the values of x for which $f(x)$ is a decreasing function. [2]

b. Sketch the graph of $y = f(x)$ showing clearly the minimum point and any asymptotic behaviour. [3]

c. Find the coordinates of the point on the graph of f where the normal to the graph is parallel to the line $y = -x$. [4]

d. This region is now rotated through 2π radians about the x-axis. Find the volume of revolution. [3]

Markscheme

a.i. attempt to use quotient rule or product rule ***M1***

$$f'(x) = \frac{\sin x\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \sqrt{x}\cos x}{\sin^2 x} \quad \left(= \frac{1}{2\sqrt{x}\sin x} - \frac{\sqrt{x}\cos x}{\sin^2 x}\right)$$ ***A1A1***

Note: Award ***A1*** for $\frac{1}{2\sqrt{x}\sin x}$ or equivalent and ***A1*** for $-\frac{\sqrt{x}\cos x}{\sin^2 x}$ or equivalent.

setting $f'(x) = 0$ ***M1***

$\frac{\sin x}{2\sqrt{x}} - \sqrt{x}\cos x = 0$

$\frac{\sin x}{2\sqrt{x}} = \sqrt{x}\cos x$ or equivalent ***A1***

$\tan x = 2x$ ***AG***

[5 marks]

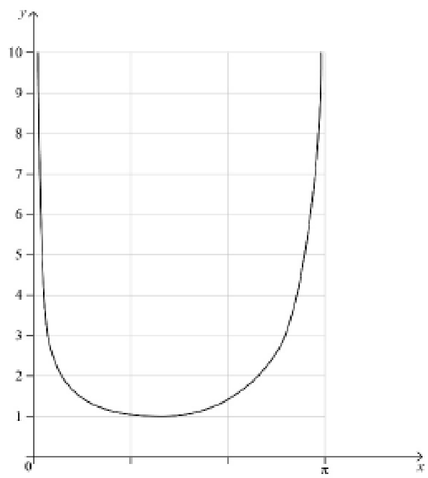
a.ii. $x = 1.17$

$0 < x \leqslant 1.17$ ***A1A1***

Note: Award ***A1*** for $0 < x$ and ***A1*** for $x \leqslant 1.17$. Accept $x < 1.17$.

[2 marks]

b.



concave up curve over correct domain with one minimum point above the x-axis. ***A1***

approaches $x = 0$ asymptotically ***A1***

approaches $x = \pi$ asymptotically ***A1***

Note: For the final ***A1*** an asymptote must be seen, and π must be seen on the x-axis or in an equation.

[3 marks]

c. $f'(x)\left(= \frac{\sin x\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \sqrt{x}\cos x}{\sin^2 x}\right) = 1$ ***(A1)***

attempt to solve for x ***(M1)***

$x = 1.96$ ***A1***

$y = f(1.96\ldots)$

$= 1.51$ ***A1***

[4 marks]

d. $V = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{x\mathrm{d}x}{\sin^2 x}$ ***(M1)(A1)***

Note: ***M1*** is for an integral of the correct squared function (with or without limits and/or π).

$= 2.68\ (= 0.852\pi)$ ***A1***

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Particle A moves such that its velocity v ms^{-1}, at time t seconds, is given by $v(t) = \frac{t}{12+t^4},\ t \geqslant 0$.

Particle B moves such that its velocity v ms^{-1} is related to its displacement s m, by the equation $v(s) = \arcsin\left(\sqrt{s}\right)$.

a. Sketch the graph of $y = v(t)$. Indicate clearly the local maximum and write down its coordinates. [2]

b. Use the substitution $u = t^2$ to find $\int \frac{t}{12+t^4}\mathrm{d}t$. [4]

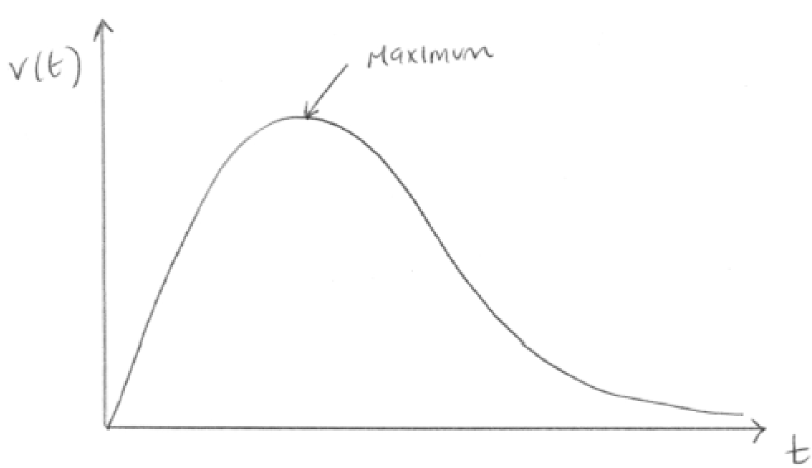
c. Find the exact distance travelled by particle A between $t = 0$ and $t = 6$ seconds. [3]

Give your answer in the form $k\arctan(b),\ k,\ b \in \mathbb{R}$.

d. Find the acceleration of particle B when $s = 0.1$ m. [3]

Markscheme

a. (a)



A1

A1 for correct shape and correct domain

$(1.41,\ 0.0884)\ \left(\sqrt{2},\ \frac{\sqrt{2}}{16}\right)$ ***A1***

[2 marks]

b. **EITHER**

$u = t^2$

$\frac{\mathrm{d}u}{\mathrm{d}t} = 2t$ ***A1***

OR

$t = u^{\frac{1}{2}}$

$\frac{\mathrm{d}t}{\mathrm{d}u} = \frac{1}{2}u^{-\frac{1}{2}}$ ***A1***

THEN

$\int \frac{t}{12+t^4}\mathrm{d}t = \frac{1}{2}\int \frac{\mathrm{d}u}{12+u^2}$ ***M1***

$= \frac{1}{2\sqrt{12}}\arctan\left(\frac{u}{\sqrt{12}}\right)(+c)$ ***M1***

$= \frac{1}{4\sqrt{3}}\arctan\left(\frac{t^2}{2\sqrt{3}}\right)(+c)$ or equivalent ***A1***

[4 marks]

c. $\int_0^6 \frac{t}{12+t^4}\mathrm{d}t$ ***(M1)***

$= \left[\frac{1}{4\sqrt{3}}\arctan\left(\frac{t^2}{2\sqrt{3}}\right)\right]_0^6$ ***M1***

$= \frac{1}{4\sqrt{3}}\left(\arctan\left(\frac{36}{2\sqrt{3}}\right)\right) \ \left(= \frac{1}{4\sqrt{3}}\left(\arctan\left(\frac{18}{\sqrt{3}}\right)\right)\right)$ (m) ***A1***

Note: Accept $\frac{\sqrt{3}}{12}\arctan\left(6\sqrt{3}\right)$ or equivalent.

[3 marks]

d. $\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{1}{2\sqrt{s(1-s)}}$ ***(A1)***

$a = v\frac{\mathrm{d}v}{\mathrm{d}s}$

$a = \arcsin\left(\sqrt{s}\right) \times \frac{1}{2\sqrt{s(1-s)}}$ ***(M1)***

$a = \arcsin\left(\sqrt{0.1}\right) \times \frac{1}{2\sqrt{0.1\times0.9}}$

$a = 0.536$ (ms^{-2}) ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Find the values of k such that the equation $x^3 + x^2 - x + 2 = k$ has three distinct real solutions.

Markscheme

from GDC, sketch a relevant graph ***A1***

maximum: $y = 3$ or (–1, 3) ***A1***

minimum: $y = 1.81$ or (0.333, 1.81) $\left(\text{or } y = \frac{49}{27} \text{ or } \left(\frac{1}{3}, \frac{49}{27}\right)\right)$ ***A1***

hence, $1.81 < k < 3$ ***A1A1*** ***N3***

Note: Award ***A1*** for $1.81 \leqslant k \leqslant 3$.

[5 marks]

Examiners report

Responses to this question were surprisingly poor. Few candidates recognised that the easier way to answer the question was to use a graph on the GDC. Many candidates embarked on fruitless algebraic manipulation which led nowhere.

When $x^2 + 4x - b$ is divided by $x - a$ the remainder is 2.

Given that $a,\ b \in \mathbb{R}$, find the smallest possible value for b.

Markscheme

$a^2 + 4a - b = 2$ ***M1A1***

EITHER

$a^2 + 4a - (b + 2) = 0$

as a is real $\Rightarrow 16 + 4(b + 2) \geqslant 0$ ***M1A1***

OR

$b = a^2 + 4a - 2$ ***M1***

$= (a + 2)^2 - 6$ ***(A1)***

THEN

$b \geqslant -6$

hence smallest possible value for b is -6 ***A1***

[5 marks]

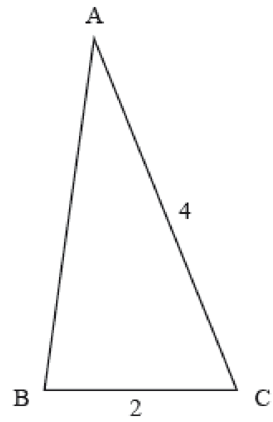
Examiners report

For quite a difficult question, there were many good solutions for this, including many different methods. It was disturbing to see how many students did not seem to be aware of the remainder theorem, instead choosing to divide the polynomial.

---

a. Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. [2]

b. The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. [4]



Markscheme

a. $k^2 - k - 12 < 0$

$(k-4)(k+3) < 0$ ***(M1)***

$-3 < k < 4$ ***A1***

[2 marks]

b. $\cos B = \frac{2^2+c^2-4^2}{4c}$ $(\text{or } 16 = 2^2 + c^2 - 4c\cos B)$ ***M1***

$\Rightarrow \frac{c^2-12}{4c} < \frac{1}{4}$ ***A1***

$\Rightarrow c^2 - c - 12 < 0$

from result in (a)

$0 < \text{AB} < 4$ or $-3 < \text{AB} < 4$ ***(A1)***

but AB must be at least 2

$\Rightarrow 2 < \text{AB} < 4$ ***A1***

Note: Allow $\leqslant \text{AB}$ for either of the final two ***A*** marks.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

The equation $x^2 - 5x - 7 = 0$ has roots α and β. The equation $x^2 + px + q = 0$ has roots $\alpha + 1$ and $\beta + 1$. Find the value of p and the value of q.

Markscheme

METHOD 1

$\alpha + \beta = 5,\ \alpha\beta = -7$ ***(M1)(A1)***

Note: Award ***M1A0*** if only one equation obtained.

$(\alpha+1)+(\beta+1)=5+2=7$ ***A1***

$(\alpha+1)(\beta+1)=\alpha\beta+(\alpha+\beta)+1$ ***(M1)***

$=-7+5+1=-1$

$p=-7,\ q=-1$ ***A1A1***

METHOD 2

$\alpha=\frac{5+\sqrt{53}}{2}=6.1\ldots;\ \beta=\frac{5-\sqrt{53}}{2}=-1.1\ldots$ ***(M1)(A1)***

$\alpha+1=\frac{7+\sqrt{53}}{2}=7.1\ldots;\ \beta+1=\frac{7-\sqrt{53}}{2}=-0.1\ldots$ ***A1***

$(x-7.14\ldots)(x+0.14\ldots)=x^2-7x-1$ ***(M1)***

$p=-7,\ q=-1$ ***A1A1***

Note: Exact answers only.

[6 marks]

Examiners report

[N/A]

Consider the function f defined by $f(x)=3x\arccos(x)$ where $-1\leqslant x\leqslant 1$.

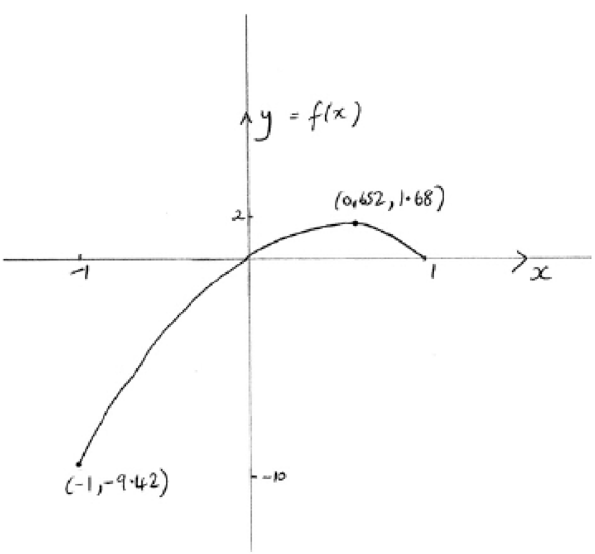
a. Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points. [3]

b. State the range of f. [2]

c. Solve the inequality $|3x\arccos(x)|>1$. [4]

Markscheme

a.



correct shape passing through the origin and correct domain ***A1***

Note: Endpoint coordinates are not required. The domain can be indicated by -1 and 1 marked on the axis.

$(0.652,\ 1.68)$ ***A1***

two correct intercepts (coordinates not required) ***A1***

Note: A graph passing through the origin is sufficient for $(0,\ 0)$.

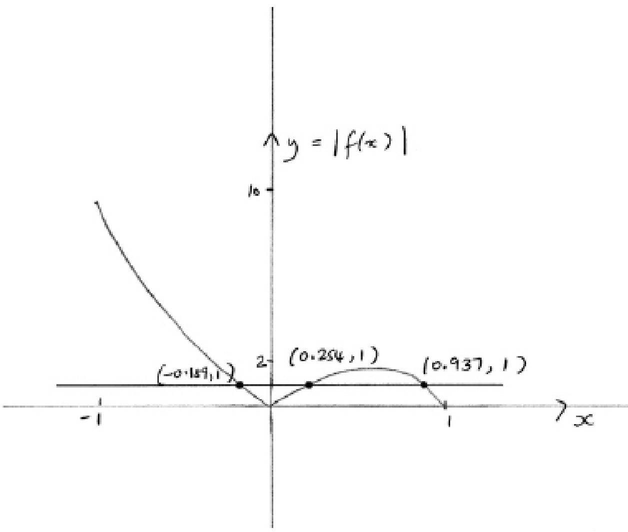
[3 marks]

b. $[-9.42,\ 1.68]\ (\text{or }-3\pi,\ 1.68])$ ***A1A1***

Note: Award ***A1A0*** for open or semi-open intervals with correct endpoints. Award ***A1A0*** for closed intervals with one correct endpoint.

[2 marks]

c. attempting to solve either $|3x\arccos(x)| > 1$ (or equivalent) or $|3x\arccos(x)| = 1$ (or equivalent) (*eg*. graphically) ***(M1)***



$x = -0.189,\ 0.254,\ 0.937$ ***(A1)***

$-1 \leqslant x < -0.189 \text{ or } 0.254 < x < 0.937$ ***A1A1***

Note: Award ***A0*** for $x < -0.189$.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A polynomial $p(x)$ with real coefficients is of degree five. The equation $p(x) = 0$ has a complex root 2 + i. The graph of $y = p(x)$ has the x-axis as a tangent at (2, 0) and intersects the coordinate axes at (−1, 0) and (0, 4).

Find $p(x)$ in factorised form with real coefficients.

Markscheme

other root is $2 - i$ ***(A1)***

a quadratic factor is therefore $(x - 2 + i)(x - 2 - i)$ ***(M1)***

$= x^2 - 4x + 5$ ***A1***

$x + 1$ is a factor ***A1***

$(x - 2)^2$ is a factor ***A1***

$p(x) = a(x + 1)(x - 2)^2(x^2 - 4x + 5)$ ***(M1)***

$p(0) = 4 \Rightarrow a = \frac{1}{5}$ ***A1***

$p(x) = \frac{1}{5}(x + 1)(x - 2)^2(x^2 - 4x + 5)$

[7 marks]

Examiners report

Whilst most candidates knew that another root was 2 − i , much fewer were able to find the quadratic factor. Surprisingly few candidates knew that $(x - 2)$ must be a repeated factor and less surprisingly many did not recognise that the whole expression needed to be multiplied by $\frac{1}{5}$.

In the quadratic equation $7x^2 - 8x + p = 0,\ (p \in \mathbb{Q})$, one root is three times the other root.

Find the value of p.

Markscheme

METHOD 1

let roots be α and 3α ***(M1)***

sum of roots $(4\alpha) = \frac{8}{7}$ ***M1***

$\Rightarrow \alpha = \frac{2}{7}$ ***A1***

EITHER

product of roots $(3\alpha^2) = \frac{p}{7}$ ***M1***

$p = 21\alpha^2 = 21 \times \frac{4}{49}$

OR

$7\left(\frac{2}{7}\right)^2 - 8\left(\frac{2}{7}\right) + p = 0$ ***M1***

$\frac{4}{7} - \frac{16}{7} + p = 0$

THEN

$\Rightarrow p = \frac{12}{7}\ (= 1.71)$ ***A1***

METHOD 2

$x = \frac{8 \pm \sqrt{64-28p}}{14}$ ***(M1)***

$\frac{8+\sqrt{64-28p}}{14} = 3\left(\frac{8-\sqrt{64-28p}}{14}\right)$ ***M1A1***

$8 + \sqrt{64-28p} = 24 - 3\sqrt{64-28p} \Rightarrow \sqrt{64-28p} = 4$ ***(M1)***

$p = \frac{12}{7}\ (= 1.71)$ ***A1***

[5 marks]

Examiners report

[N/A]

Given that the graph of $y = x^3 - 6x^2 + kx - 4$ has exactly one point at which the gradient is zero, find the value of k .

Markscheme

$\frac{dy}{dx} = 3x^2 - 12x + k$ ***M1A1***

For use of discriminant $b^2 - 4ac = 0$ or completing the square $3(x-2)^2 + k - 12$ ***(M1)***

$144 - 12k = 0$ ***(A1)***

Note: Accept trial and error, sketches of parabolas with vertex (2,0) or use of second derivative.

$k = 12$ ***A1***

[5 marks]

Examiners report

Generally candidates answer this question well using a diversity of methods. Surprisingly, a small number of candidates were successful in answering this question using the discriminant of the quadratic and in many cases reverted to trial and error to obtain the correct answer.

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a} = (3\cos\theta + 6)\boldsymbol{i} + 7\boldsymbol{j}$ and $\boldsymbol{b} = (\cos\theta - 2)\boldsymbol{i} + (1 + \sin\theta)\boldsymbol{j}$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular,

a. show that $3\sin^2\theta - 7\sin\theta + 2 = 0$; [3]

b. find the smallest possible positive value of θ. [3]

Markscheme

a. attempting to form $(3\cos\theta + 6)(\cos\theta - 2) + 7(1 + \sin\theta) = 0$ ***M1***

$3\cos^2\theta - 12 + 7\sin\theta + 7 = 0$ ***A1***

$3\left(1 - \sin^2\theta\right) + 7\sin\theta - 5 = 0$ ***M1***

$3\sin^2\theta - 7\sin\theta + 2 = 0$ ***AG***

[3 marks]

b. attempting to solve algebraically (including substitution) or graphically for $\sin\theta$ ***(M1)***

$\sin\theta = \frac{1}{3}$ ***(A1)***

$\theta = 0.340\ (= 19.5°)$ ***A1***

[3 marks]

Examiners report

a. Part (a) was very well done. Most candidates were able to use the scalar product and $\cos^2\theta = 1 - \sin^2\theta$ to show the required result.

b. Part (b) was reasonably well done. A few candidates confused 'smallest possible positive value' with a minimum function value. Some candidates gave $\theta = 0.34$ as their final answer.

The function f is of the form $f(x) = \frac{x+a}{bx+c}$, $x \ne -\frac{c}{b}$. Given that the graph of f has asymptotes $x = -4$ and $y = -2$, and that the point $\left(\frac{2}{3}, 1\right)$ lies on the graph, find the values of a , b and c .

Markscheme

vertical asymptote $x = -4 \Rightarrow -4b + c = 0$ ***M1***

horizontal asymptote $y = -2 \Rightarrow \frac{1}{b} = -2$ ***M1***

$b = -\frac{1}{2}$ and $c = -2$ ***A1A1***

$1 = \frac{\frac{2}{3}+a}{-\frac{1}{2}\times\frac{2}{3}-2}$ ***M1***

$a = -3$ ***A1***

[6 marks]

Examiners report

[N/A]

---

a. The graph of $y = \ln(x)$ is transformed into the graph of $y = \ln(2x+1)$. [2]

Describe two transformations that are required to do this.

b. Solve $\ln(2x+1) > 3\cos(x)$, $x \in [0, 10]$. [4]

Markscheme

a. **EITHER**

translation of $-\frac{1}{2}$ parallel to the x-axis

stretch of a scale factor of $\frac{1}{2}$ parallel to the x-axis ***A1A1***

OR

stretch of a scale factor of $\frac{1}{2}$ parallel to the x-axis

translation of -1 parallel to the x-axis ***A1A1***

Note: Accept clear alternative terminologies for either transformation.

[2 marks]

b. **EITHER**

$1.16 < x < 5.71 \cup 6.75 < x \leqslant 10$ ***A1A1A1A1***

OR

]1.16, 5.71[$\cup$]6.75, 10] ***A1A1A1A1***

Note: Award ***A1*** for 1 intersection value, ***A1*** for the other 2, ***A1A1*** for the intervals.

[6 marks]

Examiners report

a. This question was well done by many candidates. It would appear, however, that few candidates were aware of the standard terminology – *Stretch* and *Translation* - used to describe the relevant graph transformations. Most made good use of a GDC to find the critical points and to help in deciding on the correct intervals. A significant minority failed to note $x = 10$ as an endpoint.

b. This question was well done by many candidates. It would appear, however, that few candidates were aware of the standard terminology – *Stretch* and *Translation* - used to describe the relevant graph transformations. Most made good use of a GDC to find the critical points and to help in deciding on the correct intervals. A significant minority failed to note $x = 10$ as an endpoint.

The arithmetic sequence $\{u_n : n \in \mathbb{Z}^+\}$ has first term $u_1 = 1.6$ and common difference $d = 1.5$. The geometric sequence $\{v_n : n \in \mathbb{Z}^+\}$ has first term $v_1 = 3$ and common ratio $r = 1.2$.

a. Find an expression for $u_n - v_n$ in terms of n. [2]

b. Determine the set of values of n for which $u_n > v_n$. [3]

c. Determine the greatest value of $u_n - v_n$. Give your answer correct to four significant figures. [1]

Markscheme

a. $u_n - v_n = 1.6 + (n-1) \times 1.5 - 3 \times 1.2^{n-1}$ $(= 1.5n + 0.1 - 3 \times 1.2^{n-1})$ ***A1A1***

[2 marks]

b. attempting to solve $u_n > v_n$ numerically or graphically. ***(M1)***

$n = 2.621\ldots, 9.695\ldots$ ***(A1)***

So $3 \leqslant n \leqslant 9$ ***A1***

[3 marks]

c. The greatest value of $u_n - v_n$ is 1.642. ***A1***

Note: Do not accept 1.64.

[1 mark]

Examiners report

a. In part (a), most candidates were able to express u_n and v_n correctly and hence obtain a correct expression for $u_n - v_n$. Some candidates made careless algebraic errors when unnecessarily simplifying u_n while other candidates incorrectly stated v_n as $3(1.2)^n$.

b. In parts (b) and (c), most candidates treated *n* as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types.

c. In parts (b) and (c), most candidates treated *n* as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types. In part (c), a number of candidates attempted to find the maximum value of *n* rather than attempting to find the maximum value of $u_n - v_n$.

A particle moves in a straight line, its velocity $v\ \mathrm{ms}^{-1}$ at time t seconds is given by $v = 9t - 3t^2,\ 0 \le t \le 5$.

At time $t = 0$, the displacement s of the particle from an origin O is 3 m.

a. Find the displacement of the particle when $t = 4$. [3]

b. Sketch a displacement/time graph for the particle, $0 \le t \le 5$, showing clearly where the curve meets the axes and the coordinates of the points where the displacement takes greatest and least values. [5]

c. For $t > 5$, the displacement of the particle is given by $s = a + b\cos\frac{2\pi t}{5}$ such that s is continuous for all $t \ge 0$. [3]

Given further that $s = 16.5$ when $t = 7.5$, find the values of a and b.

d. For $t > 5$, the displacement of the particle is given by $s = a + b\cos\frac{2\pi t}{5}$ such that s is continuous for all $t \ge 0$. [4]

Find the times t_1 and $t_2 (0 < t_1 < t_2 < 8)$ when the particle returns to its starting point.

Markscheme

a. **METHOD 1**

$s = \int (9t - 3t^2)\mathrm{d}t = \frac{9}{2}t^2 - t^3(+c)$ ***(M1)***

$t = 0,\ s = 3 \Rightarrow c = 3$ ***(A1)***

$t = 4 \Rightarrow s = 11$ ***A1***

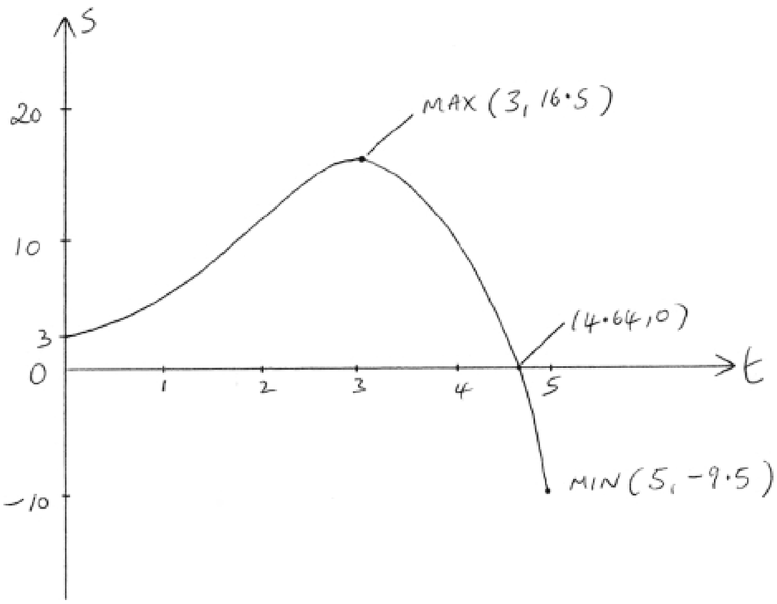
METHOD 2

$s = 3 + \int_0^4 (9t - 3t^2)\mathrm{d}t$ ***(M1)(A1)***

$s = 11$ ***A1***

[3 marks]

b.



correct shape over correct domain ***A1***

maximum at $(3,\ 16.5)$ ***A1***

t intercept at 4.64, s intercept at 3 ***A1***

minimum at $(5,\ -9.5)$ ***A1***

[5 marks]

c. $-9.5 = a + b\cos 2\pi$

$16.5 = a + b\cos 3\pi$ ***(M1)***

Note: Only award ***M1*** if two simultaneous equations are formed over the correct domain.

$a = \frac{7}{2}$ ***A1***

$b = -13$ ***A1***

[3 marks]

d. at t_1:

$3 + \frac{9}{2}t^2 - t^3 = 3$ ***(M1)***

$t^2\left(\frac{9}{2} - t\right) = 0$

$t_1 = \frac{9}{2}$ ***A1***

solving $\frac{7}{2} - 13\cos\frac{2\pi t}{5} = 3$ ***(M1)***

$\mathrm{GDC} \Rightarrow t_2 = 6.22$ ***A1***

Note: Accept graphical approaches.

[4 marks]

Total [15 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

---

a. Prove that the equation $3x^2 + 2kx + k - 1 = 0$ has two distinct real roots for all values of $k \in \mathbb{R}$. [4]

b. Find the value of k for which the two roots of the equation are closest together. [3]

Markscheme

a. $\Delta = b^2 - 4ac = 4k^2 - 4 \times 3 \times (k - 1) = 4k^2 - 12k + 12$ ***M1A1***

Note: Award ***M1A1*** if expression seen within quadratic formula.

EITHER
$144 - 4 \times 4 \times 12 < 0$ ***M1***
Δ always positive, therefore the equation always has two distinct real roots ***R1***
(and cannot be always negative as $a > 0$)
OR
sketch of $y = 4k^2 - 12k + 12$ or $y = k^2 - 3k + 3$ not crossing the x-axis ***M1***
Δ always positive, therefore the equation always has two distinct real roots ***R1***
OR
write Δ as $4(k - 1.5)^2 + 3$ ***M1***
Δ always positive, therefore the equation always has two distinct real roots ***R1***
[4 marks]

b. closest together when Δ is least ***(M1)***

minimum value occurs when $k = 1.5$ ***(M1)A1***

[3 marks]

Examiners report

a. Most candidates were able to find the discriminant (sometimes only as part of the quadratic formula) but fewer were able to explain satisfactorily why there were two distinct roots.

b. Most candidates were able to find the discriminant (sometimes only as part of the quadratic formula) but fewer were able to explain satisfactorily why there were two distinct roots. Only the better candidates were able to give good answers to part (b).

Farmer Bill owns a rectangular field, 10 m by 4 m. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

a. Given that the rope is 5 m long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer. [4]

b. Bill replaces Gruff's rope with another, this time of length $a,\ 4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field. [4]

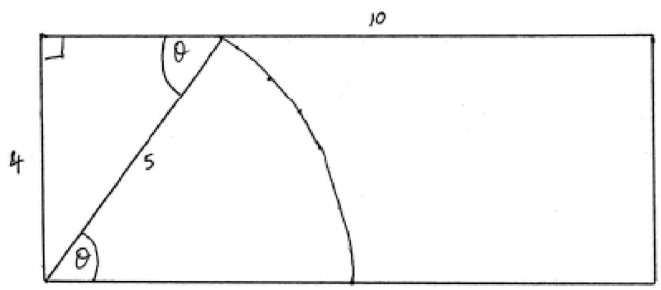
Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40.$$

c. Find the value of a. [2]

Markscheme

a.



EITHER

area of triangle $= \frac{1}{2} \times 3 \times 4 \quad (= 6)$ ***A1***

area of sector $= \frac{1}{2}\arcsin\left(\frac{4}{5}\right) \times 5^2 \quad (= 11.5911\ldots)$ ***A1***

OR

$\int_0^4 \sqrt{25 - x^2}\mathrm{d}x$ ***M1A1***

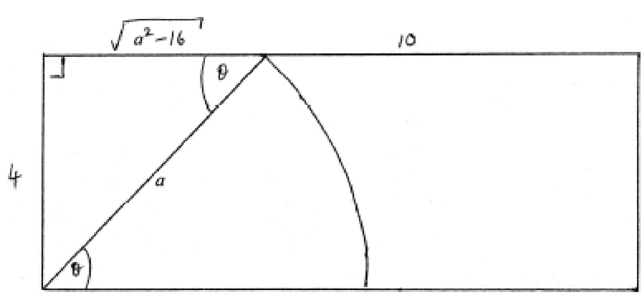
THEN

total area $= 17.5911\ldots\ \mathrm{m}^2$ ***(A1)***

percentage $= \frac{17.5911\ldots}{40} \times 100 = 44\%$ ***A1***

[4 marks]

b. **METHOD 1**



area of triangle $= \frac{1}{2} \times 4 \times \sqrt{a^2 - 16}$ ***A1***

$\theta = \arcsin\left(\frac{4}{a}\right)$ ***(A1)***

area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}a^2 \arcsin\left(\frac{4}{a}\right)$ ***A1*** therefore total area $= 2\sqrt{a^2-16} + \frac{1}{2}a^2 \arcsin\left(\frac{4}{a}\right) = 20$ ***A1***

rearrange to give: $a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2-16} = 40$ ***AG***

METHOD 2

$\int_0^4 \sqrt{a^2-x^2}\mathrm{d}x = 20$ ***M1***

use substitution $x = a\sin\theta,\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = a\cos\theta$

$\int_0^{\arcsin\left(\frac{4}{a}\right)} a^2\cos^2\theta\mathrm{d}\theta = 20$

$\frac{a^2}{2}\int_0^{\arcsin\left(\frac{4}{a}\right)} (\cos 2\theta + 1)\mathrm{d}\theta = 20$ ***M1***

$a^2\left[\left(\frac{\sin 2\theta}{2} + \theta\right)\right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$ ***A1***

$a^2[(\sin\theta\cos\theta + \theta]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$

$a^2 \arcsin\left(\frac{4}{a}\right) + a^2\left(\frac{4}{a}\right)\sqrt{\left(1-\left(\frac{4}{a}\right)^2\right)} = 40$ ***A1***

$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2-16} = 40$ ***AG***

[4 marks]

c. solving using $\mathrm{GDC} \Rightarrow a = 5.53\text{ cm}$ ***A2***

[2 marks]

Total [10 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The functions f and g are defined by

$$f(x) = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2},\ x \in \mathbb{R}$$

$$g(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2},\ x \in \mathbb{R}$$

Let $h(x) = nf(x) + g(x)$ where $n \in \mathbb{R},\ n > 1$.

Let $t(x) = \frac{g(x)}{f(x)}$.

a. (i) Show that $\frac{1}{4f(x)-2g(x)} = \frac{\mathrm{e}^x}{\mathrm{e}^{2x}+3}$. [9]

(ii) Use the substitution $u = \mathrm{e}^x$ to find $\int_0^{\ln 3} \frac{1}{4f(x)-2g(x)}\mathrm{d}x$. Give your answer in the form $\frac{\pi\sqrt{a}}{b}$ where $a,\ b \in \mathbb{Z}^+$.

b. (i) By forming a quadratic equation in e^x, solve the equation $h(x) = k$, where $k \in \mathbb{R}^+$. [8]

(ii) Hence or otherwise show that the equation $h(x) = k$ has two real solutions provided that $k > \sqrt{n^2 - 1}$ and $k \in \mathbb{R}^+$.

c. (i) Show that $t'(x) = \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ for $x \in \mathbb{R}$. [6]

(ii) Hence show that $t'(x) > 0$ for $x \in \mathbb{R}$.

Markscheme

a. (i) $\frac{1}{4\left(\frac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\right)-2\left(\frac{\mathrm{e}^x-\mathrm{e}^{-x}}{2}\right)}$ ***(M1)***

$= \frac{1}{2(\mathrm{e}^x+\mathrm{e}^{-x})-(\mathrm{e}^x-\mathrm{e}^{-x})}$ ***(A1)***

$= \frac{1}{\mathrm{e}^x+3\mathrm{e}^{-x}}$ ***A1***

$= \frac{\mathrm{e}^x}{\mathrm{e}^{2x}+3}$ ***AG***

(ii) $u = \mathrm{e}^x \Rightarrow \mathrm{d}u = \mathrm{e}^x \mathrm{d}x$ ***A1***

$\int \frac{\mathrm{e}^x}{\mathrm{e}^{2x}+3} \mathrm{d}x = \int \frac{1}{u^2+3} \mathrm{d}u$ ***M1***

(when $x = 0,\ u = 1$ and when $x = \ln 3,\ u = 3$)

$\int_1^3 \frac{1}{u^2+3} \mathrm{d}u \left[\frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right)\right]_1^3$ ***M1A1***

$\left(= \left[\frac{1}{\sqrt{3}} \arctan\left(\frac{\mathrm{e}^x}{\sqrt{3}}\right)\right]_0^{\ln 3}\right)$

$= \frac{\pi\sqrt{3}}{9} - \frac{\pi\sqrt{3}}{18}$ ***(M1)***

$= \frac{\pi\sqrt{3}}{18}$ ***A1***

[9 marks]

b. (i) $(n+1)\mathrm{e}^{2x} - 2k\mathrm{e}^x + (n-1) = 0$ ***M1A1***

$\mathrm{e}^x = \frac{2k \pm \sqrt{4k^2 - 4(n^2-1)}}{2(n+1)}$ ***M1***

$x = \ln\left(\frac{k \pm \sqrt{k^2 - n^2 + 1}}{n+1}\right)$ ***M1A1***

(ii) for two real solutions, we require $k > \sqrt{k^2 - n^2 + 1}$ ***R1***

and we also require $k^2 - n^2 + 1 > 0$ ***R1***

$k^2 > n^2 - 1$ ***A1***

$\Rightarrow k > \sqrt{n^2 - 1}$ ($k \in \mathbb{R}^+$) ***AG***

[8 marks]

c. **METHOD 1**

$t(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$

$t'(x) = \frac{(\mathrm{e}^x+\mathrm{e}^{-x})^2 - (\mathrm{e}^x-\mathrm{e}^{-x})^2}{(\mathrm{e}^x+\mathrm{e}^{-x})^2}$ ***M1A1***

$t'(x) = \frac{\left(\frac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\right)^2 - \left(\frac{\mathrm{e}^x-\mathrm{e}^{-x}}{2}\right)^2}{\left(\frac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\right)^2}$ ***A1***

$= \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ ***AG***

METHOD 2

$t'(x) = \frac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}$ ***M1A1***

$g'(x) = f(x)$ and $f'(x) = g(x)$ ***A1***

$= \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ ***AG***

METHOD 3

$t(x) = (\mathrm{e}^x - \mathrm{e}^{-x})(\mathrm{e}^x + \mathrm{e}^{-x})^{-1}$

$t'(x) = 1 - \frac{(\mathrm{e}^x - \mathrm{e}^{-x})^2}{(\mathrm{e}^x + \mathrm{e}^{-x})^2}$ ***M1A1***

$= 1 - \frac{[g(x)]^2}{[f(x)]^2}$ ***A1***

$= \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ ***AG***

METHOD 4

$t'(x) = \frac{g'(x)}{f(x)} - \frac{g(x)f'(x)}{[f(x)]^2}$ ***M1A1***

$g'(x) = f(x)$ and $f'(x) = g(x)$ gives $t'(x) = 1 - \frac{[g(x)]^2}{[f(x)]^2}$ ***A1***

$= \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ ***AG***

(ii) **METHOD 1**

$[f(x)]^2 > [g(x)]^2$ (or equivalent) ***M1A1***

$[f(x)]^2 > 0$ ***R1***

hence $t'(x) > 0,\ x \in \mathbb{R}$ ***AG***

Note: Award as above for use of either $f(x) = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$ and $g(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$ or $\mathrm{e}^x + \mathrm{e}^{-x}$ and $\mathrm{e}^x - \mathrm{e}^{-x}$.

METHOD 2

$[f(x)]^2 - [g(x)]^2 = 1$ (or equivalent) ***M1A1***

$[f(x)]^2 > 0$ ***R1***

hence $t'(x) > 0,\ x \in \mathbb{R}$ ***AG***

Note: Award as above for use of either $f(x) = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$ and $g(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$ or $\mathrm{e}^x + \mathrm{e}^{-x}$ and $\mathrm{e}^x - \mathrm{e}^{-x}$.

METHOD 3

$t'(x) = \frac{4}{(\mathrm{e}^x + \mathrm{e}^{-x})^2}$

$(\mathrm{e}^x + \mathrm{e}^{-x})^2 > 0$ ***M1A1***

$\frac{4}{(\mathrm{e}^x + \mathrm{e}^{-x})^2} > 0$ ***R1***

hence $t'(x) > 0,\ x \in \mathbb{R}$ ***AG***

[6 marks]

Examiners report

a. Parts (a) and (c) were accessible to the large majority of candidates. Candidates found part (b) considerably more challenging.

Part (a)(i) was reasonably well done with most candidates able to show that $\frac{1}{4f(x) - 2g(x)} = \frac{\mathrm{e}^x}{\mathrm{e}^{2x} + 3}$. In part (a)(ii), a number of candidates correctly used the required substitution to obtain $\int \frac{\mathrm{e}^x}{\mathrm{e}^{2x}+3}\mathrm{d}x = \int \frac{1}{u^2+3}\mathrm{d}u$ but then thought that the antiderivative involved natural log rather than arctan.

b. Parts (a) and (c) were accessible to the large majority of candidates. Candidates found part (b) considerably more challenging.

In part (b)(i), a reasonable number of candidates were able to form a quadratic in e^x (involving parameters n and k) and then make some progress towards solving for e^x in terms of n and k. Having got that far, a small number of candidates recognised to then take the natural logarithm of both sides and hence solve $h(x) = k$ for χ. In part (b)(ii), a small number of candidates were able to show from their solutions to part (b)(i) or through the use of the discriminant that the equation $h(x) = k$ has two real solutions provided that $k > \sqrt{k^2 - n^2 + 1}$ and $k > \sqrt{n^2 - 1}$.

c. Parts (a) and (c) were accessible to the large majority of candidates. Candidates found part (b) considerably more challenging.

It was pleasing to see the number of candidates who attempted part (c). In part (c)(i), a large number of candidates were able to correctly apply either the quotient rule or the product rule to find $t'(x)$. A smaller number of candidates were then able to show equivalence between the form of $t'(x)$ they had obtained and the form of $t'(x)$ required in the question. A pleasing number of candidates were able to exploit the property that $f'(x) = g(x)$ and $g'(x) = f(x)$. As with part (c)(i), part (c)(ii) could be successfully tackled in a number of ways. The best candidates offered concise logical reasoning to show that $t'(x) > 0$ for $x \in \mathbb{R}$.

Let the function f be defined by $f(x) = \frac{2-\mathrm{e}^x}{2\mathrm{e}^x-1}$, $x \in D$.

a. Determine D, the largest possible domain of f. [2]

b. Show that the graph of f has three asymptotes and state their equations. [5]

c. Show that $f'(x) = -\frac{3\mathrm{e}^x}{(2\mathrm{e}^x-1)^2}$. [3]

d. Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. [4]

e. Find an expression for $f^{-1}(x)$. [4]

f. Consider the region R enclosed by the graph of $y = f(x)$ and the axes. [4]

Find the volume of the solid obtained when R is rotated through 2π about the y-axis.

Markscheme

a. attempting to solve either $2\mathrm{e}^x - 1 = 0$ or $2\mathrm{e}^x - 1 \neq 0$ for x ***(M1)***

$D = \mathbb{R} \backslash \{-\ln 2\}$ (or equivalent *eg* $x \neq -\ln 2$) ***A1***

Note: Accept $D = \mathbb{R} \backslash \{-0.693\}$ or equivalent *eg* $x \neq -0.693$.

[2 marks]

b. considering $\lim\limits_{x \to -\ln 2} f(x)$ ***(M1)***

$x = -\ln 2$ $(x = -0.693)$ ***A1***

considering one of $\lim\limits_{x \to -\infty} f(x)$ or $\lim\limits_{x \to +\infty} f(x)$ ***M1***

$\lim\limits_{x \to -\infty} f(x) = -2 \Rightarrow y = -2$ ***A1***

$\lim\limits_{x \to +\infty} f(x) = -\frac{1}{2} \Rightarrow y = -\frac{1}{2}$ ***A1***

Note: Award ***A0A0*** for $y = -2$ and $y = -\frac{1}{2}$ stated without any justification.

[5 marks]

c. $f'(x) = \frac{-e^x(2e^x-1)-2e^x(2-e^x)}{(2e^x-1)^2}$ ***M1A1A1***

$= -\frac{3e^x}{(2e^x-1)^2}$ ***AG***

[3 marks]

d. $f'(x) < 0$ (for all $x \in D$) $\Rightarrow f$ is (strictly) decreasing ***R1***

Note: Award ***R1*** for a statement such as $f'(x) \neq 0$ and so the graph of f has no turning points.

one branch is above the upper horizontal asymptote and the other branch is below the lower horizontal asymptote ***R1***

f has an inverse ***AG***

$-\infty < x < -2 \cup -\frac{1}{2} < x < \infty$ ***A2***

Note: Award ***A2*** if the domain of the inverse is seen in either part (d) or in part (e).

[4 marks]

e. $x = \frac{2-e^y}{2e^y-1}$ ***M1***

Note: Award ***M1*** for interchanging x and y (can be done at a later stage).

$2xe^y - x = 2 - e^y$ ***M1***

$e^y(2x+1) = x + 2$ ***A1***

$f^{-1}(x) = \ln\left(\frac{x+2}{2x+1}\right)$ $\left(f^{-1}(x) = \ln(x+2) - \ln(2x+1)\right)$ ***A1***

[4 marks]

f. use of $V = \pi \int_a^b x^2 \mathrm{d}y$ ***(M1)***

$= \pi \int_0^1 \left(\ln\left(\frac{y+2}{2y+1}\right)\right)^2 \mathrm{d}y$ ***(A1)(A1)***

Note: Award ***(A1)*** for the correct integrand and ***(A1)*** for the limits.

$= 0.331$ ***A1***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
[N/A]

e. [N/A]
f. [N/A]

The graphs of $y = x^2\mathrm{e}^{-x}$ and $y = 1 - 2\sin x$ for $2 \leqslant x \leqslant 7$ intersect at points A and B.

The x-coordinates of A and B are x_{A} and x_{B}.

a. Find the value of x_{A} and the value of x_{B}. [2]

b. Find the area enclosed between the two graphs for $x_{\mathbf{A}} \leqslant x \leqslant x_{\mathrm{B}}$. [3]

Markscheme

a. $x_{\mathrm{A}} = 2.87$ ***A1***

$x_{\mathrm{B}} = 6.78$ ***A1***

[2 marks]

b. $\int_{2.87172\mathrm{K}}^{6.77681K} 1 - 2\sin x - x^2\mathrm{e}^{-x}\mathrm{d}x$ ***(M1)(A1)***

$= 6.76$ ***A1***

Note: Award ***(M1)*** for definite integral and ***(A1***) for a correct definite integral.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

(a) Find the solution of the equation

$$\ln 2^{4x-1} = \ln 8^{x+5} + \log_2 16^{1-2x},$$

expressing your answer in terms of $\ln 2$.

(b) Using this value of x, find the value of a for which $\log_a x = 2$, giving your answer to three decimal places.

Markscheme

(a) rewrite the equation as $(4x-1)\ln 2 = (x+5)\ln 8 + (1-2x)\log_2 16$ ***(M1)***

$(4x-1)\ln 2 = (3x+15)\ln 2 + 4 - 8x$ ***(M1)(A1)***

$x = \frac{4+16\ln 2}{8+\ln 2}$ ***A1***

(b) $x = a^2$ ***(M1)***

$a = 1.318$ ***A1***

Note: Treat 1.32 as an ***AP***.

Award ***A0*** for $\pm$.

[6 marks]

Examiners report

A more difficult question. Many candidates failed to read the question carefully so did not express x in terms of $\ln 2$.

Given that $(x - 2)$ is a factor of $f(x) = x^3 + ax^2 + bx - 4$ and that division $f(x)$ by $(x - 1)$ leaves a remainder of -6 , find the value of a and the value of b .

Markscheme

$f(2) = 8 + 4a + 2b - 4 = 0$ ***M1***

$\Rightarrow 4a + 2b = -4$ ***A1***

$f(1) = 1 + a + b - 4 = -6$ ***M1***

$\Rightarrow a + b = -3$ ***A1***

solving, $a = 1,\ b = -4$ ***A1A1***

[6 marks]

Examiners report

[N/A]

Given that $f(x) = \frac{1}{1+e^{-x}}$,

a. find $f^{-1}(x)$, stating its domain; [6]

b. find the value of x such that $f(x) = f^{-1}(x)$. [1]

Markscheme

a. $y = \frac{1}{1+e^{-x}}$

$y(1 + e^{-x}) = 1$ ***M1***

$1 + e^{-x} = \frac{1}{y} \Rightarrow e^{-x} = \frac{1}{y} - 1$ ***A1***

$\Rightarrow x = -\ln\left(\frac{1}{y} - 1\right)$ ***A1***

$f^{-1}(x) = -\ln\left(\frac{1}{x} - 1\right) \quad \left(= \ln\left(\frac{x}{1-x}\right)\right)$ ***A1***

domain: $0 < x < 1$ ***A1A1***

Note: Award ***A1*** for endpoints and ***A1*** for strict inequalities.

[6 marks]

b. 0.659 ***A1***

[1 mark]

Examiners report

a. Finding the inverse function was done successfully by a very large number of candidates. The domain, however, was not always correct. Some candidates failed to use the GDC correctly to find (b), while other candidates had unsuccessful attempts at an analytic solution.

b. Finding the inverse function was done successfully by a very large number of candidates. The domain, however, was not always correct. Some candidates failed to use the GDC correctly to find (b), while other candidates had unsuccessful attempts at an analytic solution.

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.

a. Find the first term and the common difference. [4]

b. Find the smallest value of *n* such that the sum of the first *n* terms is greater than 600. [3]

Markscheme

a. $S_n = \frac{n}{2}[2a + (n-1)d]$

$212 = \frac{16}{2}(2a + 15d) \quad (= 16a + 120d)$ ***A1***

n^{th} term is $a + (n-1)d$

$8 = a + 4d$ ***A1***

solving simultaneously: ***(M1)***

$d = 1.5,\ a = 2$ ***A1***

[4 marks]

b. $\frac{n}{2}[4 + 1.5(n-1)] > 600$ ***(M1)***

$\Rightarrow 3n^2 + 5n - 2400 > 0$ ***(A1)***

$\Rightarrow n > 27.4\ldots,\ (n < -29.1\ldots)$

Note: Do not penalize improper use of inequalities.

$\Rightarrow n = 28$ ***A1***

[3 marks]

Examiners report

a. This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

b. This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

The function f is defined as $f(x) = \sqrt{\frac{1-x}{1+x}}, \quad -1 < x \leqslant 1.$

Find the inverse function, f^{-1} stating its domain and range.

Markscheme

$x = \sqrt{\frac{1-y}{1+y}}$ ***M1***

Note: Award ***M1*** for interchanging x and y (can be done at a later stage).

$x^2 = \frac{1-y}{1+y}$

$x^2 + x^2y = 1 - y$ ***M1***

Note: Award ***M1*** for attempting to make y the subject.

$y(1 + x^2) = 1 - x^2$ ***(A1)***

$f^{-1}(x) = \frac{1-x^2}{1+x^2}, \ x \geqslant 0$ ***A1A1***

Note: Award ***A1*** only if $f^{-1}(x)$ is seen. Award ***A1*** for the domain.

the range of f^{-1} is $-1 < f^{-1}(x) \leqslant 1$ ***A1***

Note: Accept correct alternative notation *eg*. $-1 < y \leqslant 1$.

[6 marks]

Examiners report

Most candidates were able to find an expression for the inverse function. A large number of candidates however were unable to determine the domain and range of the inverse.

Compactness is a measure of how compact an enclosed region is.

The compactness, C , of an enclosed region can be defined by $C = \frac{4A}{\pi d^2}$, where A is the area of the region and d is the maximum distance between any two points in the region.

For a circular region, $C = 1$.

Consider a regular polygon of n sides constructed such that its vertices lie on the circumference of a circle of diameter x units.

a. If $n > 2$ and even, show that $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$. [3]

b. If $n > 1$ and odd, it can be shown that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$. [4]

Find the regular polygon with the least number of sides for which the compactness is more than 0.99.

c. If $n > 1$ and odd, it can be shown that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$. [1]

Comment briefly on whether *C* is a good measure of compactness.

Markscheme

a. each triangle has area $\frac{1}{8}x^2\sin\frac{2\pi}{n}$ $\quad$ (use of $\frac{1}{2}ab\sin C$) ***(M1)***

there are n triangles so $A = \frac{1}{8}nx^2\sin\frac{2\pi}{n}$ ***A1***

$C = \frac{4\left(\frac{1}{8}nx^2\sin\frac{2\pi}{n}\right)}{\pi n^2}$ ***A1***

so $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$ ***AG***

[3 marks]

b. attempting to find the least value of n such that $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ ***(M1)***

$n = 26$ ***A1***

attempting to find the least value of n such that $\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ ***(M1)***

$n = 21$ (and so a regular polygon with 21 sides) ***A1***

Note: Award ***(M0)A0(M1)A1*** if $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ is not considered and $\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ is correctly considered.

Award ***(M1)A1(M0)A0*** for $n = 26$.

[4 marks]

c. **EITHER**

for even and odd values of *n*, the value of *C* seems to increase towards the limiting value of the circle $(C = 1)$ *ie* as *n* increases, the polygonal regions get closer and closer to the enclosing circular region ***R1***

OR

the differences between the odd and even values of *n* illustrate that this measure of compactness is not a good one. ***R1***

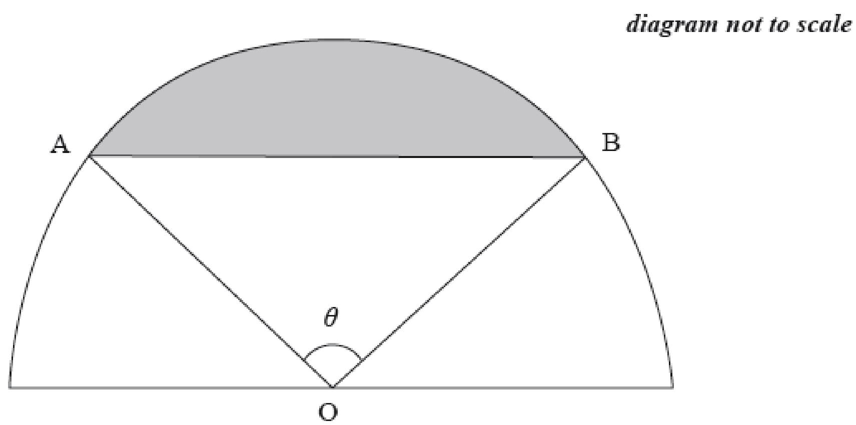
Examiners report

a. Most candidates found this a difficult question with a large number of candidates either not attempting it or making little to no progress. In part (a), a number of candidates attempted to show the desired result using specific regular polygons. Some candidates attempted to fudge the result.

b. In part (b), the overwhelming majority of candidates that obtained either $n = 21$ or $n = 26$ or both used either a GDC numerical solve feature or a graphical approach rather than a tabular approach which is more appropriate for a discrete variable such as the number of sides of a regular polygon. Some candidates wasted valuable time by showing that $C = \frac{n \sin \frac{2\pi}{n}}{\pi\left(1+\cos \frac{\pi}{n}\right)}$ (a given result).

c. In part (c), the occasional candidate correctly commented that C was a good measure of compactness either because the value of C seemed to approach the limiting value of the circle as n increased or commented that C was not a good measure because of the disparity in C-values between even and odd values of n.

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that $\mathrm{A\hat{O}B} = \theta$, where θ is in radians.



a. Show that the shaded area can be expressed as $50\theta - 50\sin\theta$. [2]

b. Find the value of θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures. [3]

Markscheme

a. $A = \frac{1}{2} \times 10^2 \times \theta - \frac{1}{2} \times 10^2 \times \sin\theta$ ***M1A1***

Note: Award ***M1*** for use of area of segment = area of sector – area of triangle.

$= 50\theta - 50\sin\theta$ ***AG***

[2 marks]

b. **METHOD 1**

unshaded area $= \frac{\pi \times 10^2}{2} - 50(\theta - \sin\theta)$

(or equivalent *eg* $50\pi - 50\theta + 50\sin\theta$) ***(M1)***

$50\theta - 50\sin\theta = \frac{1}{2}(50\pi - 50\theta + 50\sin\theta)$ ***(A1)***

$3\theta - 3\sin\theta - \pi = 0$

$\Rightarrow \theta = 1.969$ (rad) ***A1***

METHOD 2

$50\theta - 50\sin\theta = \frac{1}{3}\left(\frac{\pi \times 10^2}{2}\right)$ ***(M1)(A1)***

$3\theta - 3\sin\theta - \pi = 0$

$\Rightarrow \theta = 1.969 \text{ (rad)}$ ***A1***

[3 marks]

Examiners report

a. Part (a) was very well done. Most candidates knew how to calculate the area of a segment. A few candidates used $r = 20$.

b. Part (b) challenged a large proportion of candidates. A common error was to equate the unshaded area and the shaded area. Some candidates expressed their final answer correct to three significant figures rather than to the four significant figures specified.

Let $f(x) = \ln x$. The graph of *f* is transformed into the graph of the function *g* by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the *x*-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

Markscheme

$h(x) = f(x-3) - 2 = \ln(x-3) - 2$ ***(M1)(A1)***

$g(x) = -h(x) = 2 - \ln(x-3)$ ***M1***

Note*:* Award **M1** only if it is clear the effect of the reflection in the *x*-axis:

the expression is correct ***OR***
there is a change of signs of the previous expression ***OR***
there's a graph or an explanation making it explicit

$= \ln e^2 - \ln(x-3)$ ***M1***

$= \ln\left(\frac{e^2}{x-3}\right)$ ***A1***

[5 marks]

Examiners report

This question was well attempted but many candidates could have scored better had they written down all the steps to obtain the final expression. In some cases, as the final expression was incorrect and the middle steps were missing, candidates scored just 1 mark. That could be a consequence of a small mistake, but the lack of working prevented them from scoring at least all method marks. Some candidates performed the transformations well but were not able to use logarithms properties to transform the answer and give it as a single logarithm.

Find the acute angle between the planes with equations $x + y + z = 3$ and $2x - z = 2$.

Markscheme

$\boldsymbol{n}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\boldsymbol{n}_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ ***(A1)(A1)***

EITHER

$\theta = \arccos\left(\frac{n_1 \bullet n_2}{|n_1||n_2|}\right) \left(\cos\theta = \frac{n_1 \bullet n_2}{|n_1||n_2|}\right)$ ***(M1)***

$= \arccos\left(\frac{2+0-1}{\sqrt{3}\sqrt{5}}\right) \left(\cos\theta = \frac{2+0-1}{\sqrt{3}\sqrt{5}}\right)$ ***(A1)***

$= \arccos\left(\frac{1}{\sqrt{15}}\right) \left(\cos\theta = \frac{1}{\sqrt{15}}\right)$

OR

$\theta = \arcsin\left(\frac{|n_1 \times n_2|}{|n_1||n_2|}\right) \left(\sin\theta = \frac{|n_1 \times n_2|}{|n_1||n_2|}\right)$ ***(M1)***

$= \arcsin\left(\frac{\sqrt{14}}{\sqrt{3}\sqrt{5}}\right) \left(\sin\theta = \frac{\sqrt{14}}{\sqrt{3}\sqrt{5}}\right)$ ***(A1)***

$= \arcsin\left(\frac{\sqrt{14}}{\sqrt{15}}\right) \left(\sin\theta = \frac{\sqrt{14}}{\sqrt{15}}\right)$

THEN

$= 75.0^\circ$ (or 1.31) ***A1***

[5 marks]

Examiners report

[N/A]

Let $f(x) = \frac{e^{2x}+1}{e^x-2}$.

The line L_2 is parallel to L_1 and tangent to the curve $y = f(x)$.

a. Find the equations of the horizontal and vertical asymptotes of the curve $y = f(x)$. [4]

b. (i) Find $f'(x)$. [8]

(ii) Show that the curve has exactly one point where its tangent is horizontal.

(iii) Find the coordinates of this point.

c. Find the equation of L_1, the normal to the curve at the point where it crosses the y-axis. [4]

d. Find the equation of the line L_2. [5]

Markscheme

a. $x \to -\infty \Rightarrow y \to -\frac{1}{2}$ so $y = -\frac{1}{2}$ is an asymptote ***(M1)A1***

$e^x - 2 = 0 \Rightarrow x = \ln 2$ so $x = \ln 2\ (= 0.693)$ is an asymptote ***(M1)A1***

[4 marks]

b. (i) $f'(x) = \frac{2(e^x-2)e^{2x}-(e^{2x}+1)e^x}{(e^x-2)^2}$ ***M1A1***

$= \frac{e^{3x}-4e^{2x}-e^x}{(e^x-2)^2}$

(ii) $f'(x) = 0$ when $e^{3x} - 4e^{2x} - e^x = 0$ ***M1***

$e^x\left(e^{2x} - 4e^x - 1\right) = 0$

$e^x = 0, \ e^x = -0.236, \ e^x = 4.24$ (or $e^x = 2 \pm \sqrt{5}$) ***A1A1***

Note: Award ***A1*** for zero, ***A1*** for other two solutions.
Accept any answers which show a zero, a negative and a positive.

as $e^x > 0$ exactly one solution ***R1***

Note: Do not award marks for purely graphical solution.

(iii) (1.44, 8.47) ***A1A1***

[8 marks]

c. $f'(0) = -4$ ***(A1)***

so gradient of normal is $\frac{1}{4}$ ***(M1)***

$f(0) = -2$ ***(A1)***

so equation of L_1 is $y = \frac{1}{4}x - 2$ ***A1***

[4 marks]

d. $f'(x) = \frac{1}{4}$ ***M1***

so $x = 1.46$ ***(M1)A1***

$f(1.46) = 8.47$ ***(A1)***

equation of L_2 is $y - 8.47 = \frac{1}{4}(x - 1.46)$ ***A1***

(or $y = \frac{1}{4}x + 8.11$)

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

---

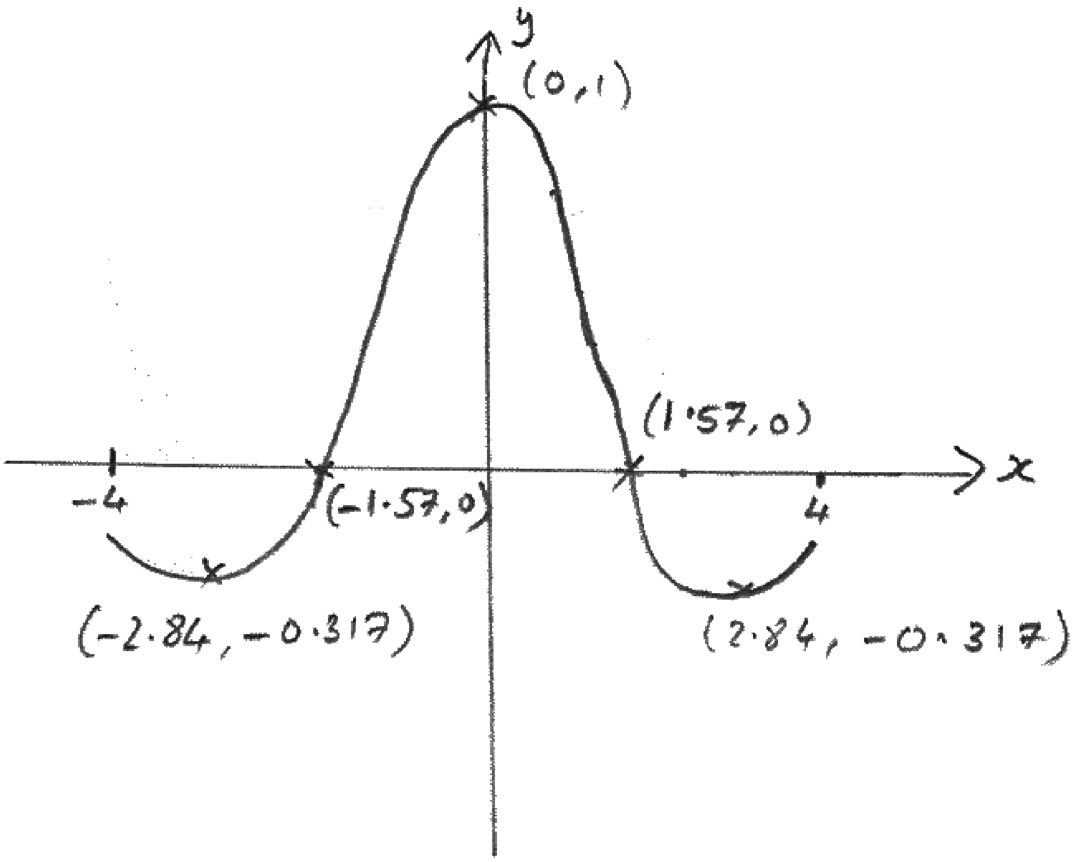
a. Sketch the curve $y = \frac{\cos x}{\sqrt{x^2+1}}, \ -4 \leqslant x \leqslant 4$ showing clearly the coordinates of the x-intercepts, any maximum points and any minimum points. [4]

b. Write down the gradient of the curve at $x = 1$. [1]

c. Find the equation of the normal to the curve at $x = 1$. [3]

Markscheme

a.



A1A1A1A1

Note: Award ***A1*** for correct shape. Do not penalise if too large a domain is used,
A1 for correct x-intercepts,
A1 for correct coordinates of two minimum points,
A1 for correct coordinates of maximum point.
Accept answers which correctly indicate the position of the intercepts, maximum point and minimum points.

[4 marks]

b. gradient at $x = 1$ is -0.786 ***A1***

[1 mark]

c. gradient of normal is $\frac{-1}{-0.786}(= 1.272...)$ ***(A1)***

when $x = 1$, $y = 0.3820...$ ***(A1)***
Equation of normal is $y - 0.382 = 1.27(x - 1)$ ***A1***
$(\Rightarrow y = 1.27x - 0.890)$
[3 marks]

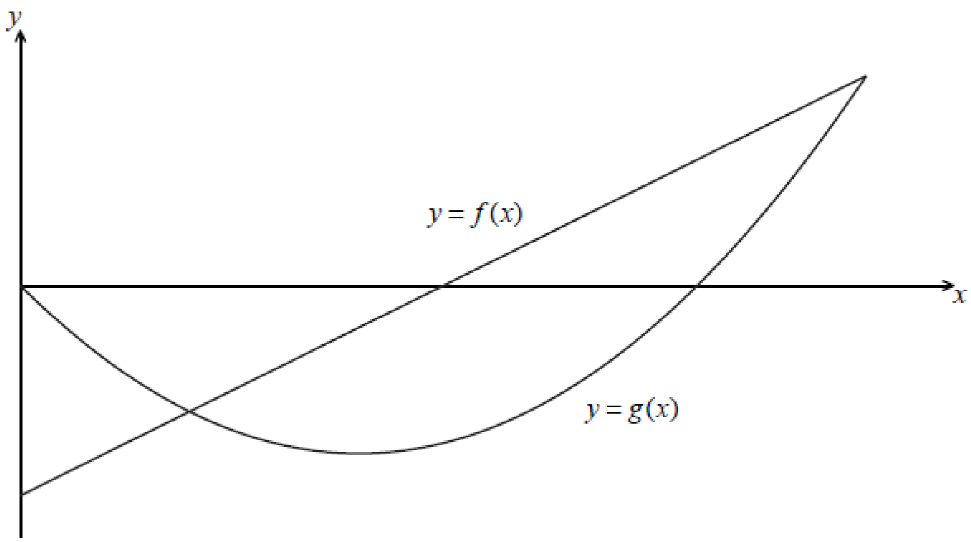
Examiners report

a. Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

b. Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

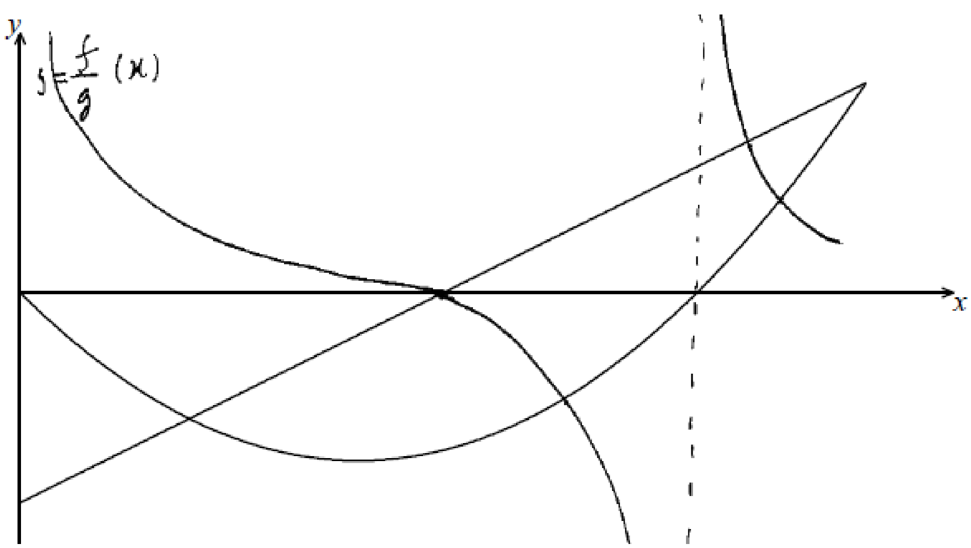
c. Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

The diagram shows the graphs of a linear function f and a quadratic function g.



On the same axes sketch the graph of $\frac{f}{g}$. Indicate clearly where the x-intercept and the asymptotes occur.

Markscheme



correct concavities ***A1A1***

Note: Award ***A1*** for concavity of each branch of the curve.

correct x-intercept of $\frac{f}{g}$ (which is EXACTLY the x-intercept of f) ***A1***

correct vertical asymptotes of $\frac{f}{g}$ (which ONLY occur when x equals the x-intercepts of g) ***A1A1***

[5 marks]

Examiners report

Many candidates answered well this question. Full marks were often achieved. Many other candidates did not attempt it at all.

---

a. (i) Express the sum of the first n positive odd integers using sigma notation. [4]

(ii) Show that the sum stated above is n^2.

(iii) Deduce the value of the difference between the sum of the first 47 positive odd integers and the sum of the first 14 positive odd integers.

b. A number of distinct points are marked on the circumference of a circle, forming a polygon. Diagonals are drawn by joining all pairs of non-adjacent points. [7]

(i) Show on a diagram all diagonals if there are 5 points.

(ii) Show that the number of diagonals is $\frac{n(n-3)}{2}$ if there are n points, where $n > 2$.

(iii) Given that there are more than one million diagonals, determine the least number of points for which this is possible.

c. The random variable $X \sim B(n,\ p)$ has mean 4 and variance 3. [8]

(i) Determine n and p.

(ii) Find the probability that in a single experiment the outcome is 1 or 3.

Markscheme

a. (i) $\sum\limits_{k=1}^{n}(2k-1)$ (or equivalent) ***A1***

Note: Award ***A0*** for $\sum\limits_{n=1}^{n}(2n-1)$ or equivalent.

(ii) **EITHER**

$2 \times \frac{n(n+1)}{2} - n$ ***M1A1***

OR

$\frac{n}{2}(2+(n-1)2)$ (using $S_n = \frac{n}{2}(2u_1+(n-1)d)$) ***M1A1***

OR

$\frac{n}{2}(1+2n-1)$ (using $S_n = \frac{n}{2}(u_1+u_n)$) ***M1A1***

THEN

$= n^2$ ***AG***

(iii) $47^2 - 14^2 = 2013$ ***A1***

[4 marks]

b. (i) **EITHER**

a pentagon and five diagonals ***A1***

OR

five diagonals (circle optional) ***A1***

(ii) Each point joins to $n - 3$ other points. ***A1***

a correct argument for $n(n-3)$ ***R1***

a correct argument for $\frac{n(n-3)}{2}$ ***R1***

(iii) attempting to solve $\frac{1}{2}n(n-3) > 1\,000\,000$ for n. ***(M1)***

$n > 1415.7$ ***(A1)***

$n = 1416$ ***A1***

[7 marks]

c. (i) $np = 4$ and $npq = 3$ ***(A1)***

attempting to solve for n and p ***(M1)***

$n = 16$ and $p = \frac{1}{4}$ ***A1***

(ii) $X \sim B(16, 0.25)$ ***(A1)***

$P(X = 1) = 0.0534538...(= \binom{16}{1}(0.25)(0.75)^{15})$ ***(A1)***

$P(X = 3) = 0.207876...(= \binom{16}{3}(0.25)^3(0.75)^{13})$ ***(A1)***

$\mathrm{P}(X = 1) + \mathrm{P}(X = 3)$ ***(M1)***

$= 0.261$ ***A1***

[8 marks]

Examiners report

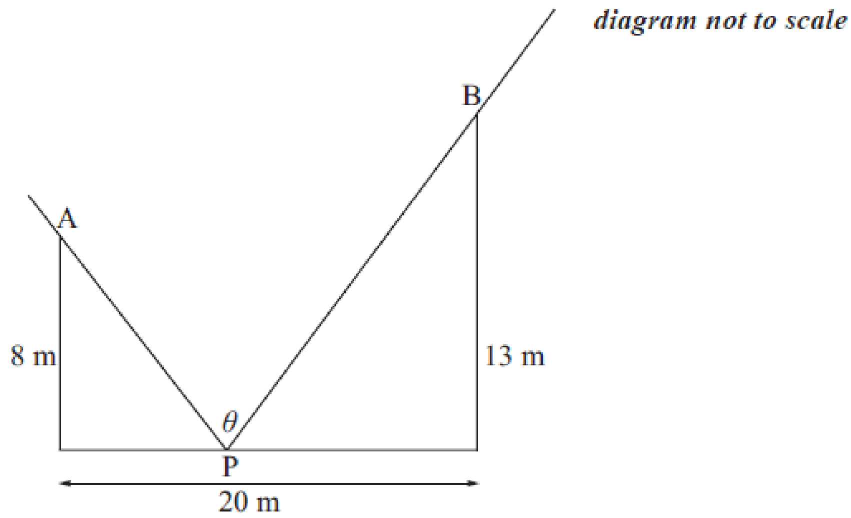
a. In part (a) (i), a large number of candidates were unable to correctly use sigma notation to express the sum of the first n positive odd integers. Common errors included summing $2n - 1$ from 1 to n and specifying sums with incorrect limits. Parts (a) (ii) and (iii) were generally well done.

b. Parts (b) (i) and (iii) were generally well done. In part (b) (iii), many candidates unnecessarily simplified their quadratic when direct GDC use could have been employed. A few candidates gave $n > 1416$ as their final answer. While some candidates displayed sound reasoning in part (b) (ii), many candidates unfortunately adopted a ‘proof by example’ approach.

c. Part (c) was generally well done. In part (c) (ii), some candidates multiplied the two probabilities rather than adding the two probabilities.

A straight street of width 20 metres is bounded on its parallel sides by two vertical walls, one of height 13 metres, the other of height 8 metres.

The intensity of light at point P at ground level on the street is proportional to the angle θ where $\theta = \mathrm{A\hat{P}B}$, as shown in the diagram.



a. Find an expression for θ in terms of x, where x is the distance of P from the base of the wall of height 8 m. [2]

b. (i) Calculate the value of θ when $x = 0$. [2]

(ii) Calculate the value of θ when $x = 20$.

c. Sketch the graph of θ, for $0 \leqslant x \leqslant 20$. [2]

d. Show that $\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$. [6]

e. Using the result in part (d), or otherwise, determine the value of x corresponding to the maximum light intensity at P. Give your answer to four significant figures. [3]

f. The point P moves across the street with speed 0.5 ms^{-1}. Determine the rate of change of θ with respect to time when P is at the midpoint of the street. [4]

Markscheme

a. **EITHER**

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent) ***M1A1***

Note: Accept $\theta = 180^\circ - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent).

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ (or equivalent) ***M1A1***

[2 marks]

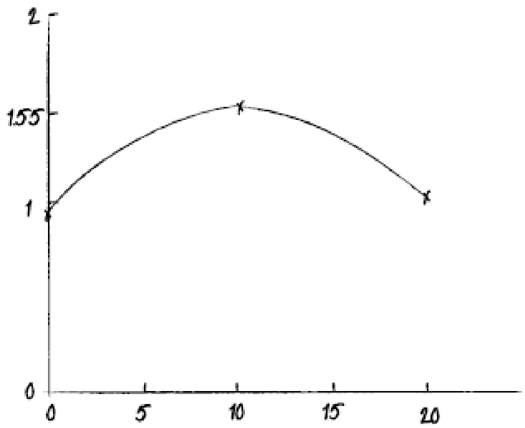
b. (i) $\theta = 0.994$ $\left(= \arctan\frac{20}{13}\right)$ ***A1***

(ii) $\theta = 1.19$ $\left(= \arctan\frac{5}{2}\right)$ ***A1***

[2 marks]

c. correct shape. ***A1***

correct domain indicated. ***A1***



[2 marks]

d. attempting to differentiate one $\arctan(f(x))$ term ***M1***

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{8}{x^2} \times \frac{1}{1+\left(\frac{8}{x}\right)^2} - \frac{13}{(20-x)^2} \times \frac{1}{1+\left(\frac{13}{20-x}\right)^2}$ ***A1A1***

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\frac{1}{8}}{1+\left(\frac{x}{8}\right)^2} + \frac{-\frac{1}{13}}{1+\left(\frac{20-x}{13}\right)^2}$ ***A1A1***

THEN

$= \frac{8}{x^2+64} - \frac{13}{569-40x+x^2}$ ***A1***

$= \frac{8(569-40x+x^2)-13(x^2+64)}{(x^2+64)(x^2-40x+569)}$ ***M1A1***

$= \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$ ***AG***

[6 marks]

e. Maximum light intensity at P occurs when $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0$. ***(M1)***

either attempting to solve $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0$ for *x* or using the graph of either θ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$ ***(M1)***

x = 10.05 (m) ***A1***

[3 marks]

f. $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.5$ ***(A1)***

At *x* = 10, $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0.000453 \left(= \frac{5}{11029}\right)$. ***(A1)***

use of $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0.000227 \left(= \frac{5}{22058}\right) (\text{rad s}^{-1})$ ***A1***

Note: Award ***(A1)*** for $\frac{\mathrm{d}x}{\mathrm{d}t} = -0.5$ and ***A1*** for $\frac{\mathrm{d}\theta}{\mathrm{d}t} = -0.000227 \left(= -\frac{5}{22058}\right)$.

Note: Implicit differentiation can be used to find $\frac{\mathrm{d}\theta}{\mathrm{d}t}$. Award as above.

[4 marks]

Examiners report

a. Part (a) was reasonably well done. While many candidates exhibited sound trigonometric knowledge to correctly express θ in terms of x, many other candidates were not able to use elementary trigonometry to formulate the required expression for θ.

b. In part (b), a large number of candidates did not realize that θ could only be acute and gave obtuse angle values for θ. Many candidates also demonstrated a lack of insight when substituting endpoint x-values into θ.

c. In part (c), many candidates sketched either inaccurate or implausible graphs.

d. In part (d), a large number of candidates started their differentiation incorrectly by failing to use the chain rule correctly.

e. For a question part situated at the end of the paper, part (e) was reasonably well done. A large number of candidates demonstrated a sound knowledge of finding where the maximum value of θ occurred and rejected solutions that were not physically feasible.

f. In part (f), many candidates were able to link the required rates, however only a few candidates were able to successfully apply the chain rule in a related rates context.

A particle, A, is moving along a straight line. The velocity, $v_A\ \mathrm{ms}^{-1}$, of A t seconds after its motion begins is given by

$$v_A = t^3 - 5t^2 + 6t.$$

a. Sketch the graph of $v_A = t^3 - 5t^2 + 6t$ for $t \geqslant 0$, with v_A on the vertical axis and t on the horizontal. Show on your sketch the local maximum and minimum points, and the intercepts with the t-axis. [3]

b. Write down the times for which the velocity of the particle is increasing. [2]

c. Write down the times for which the magnitude of the velocity of the particle is increasing. [3]

d. At $t = 0$ the particle is at point O on the line. [3]

   Find an expression for the particle's displacement, x_Am, from O at time t.

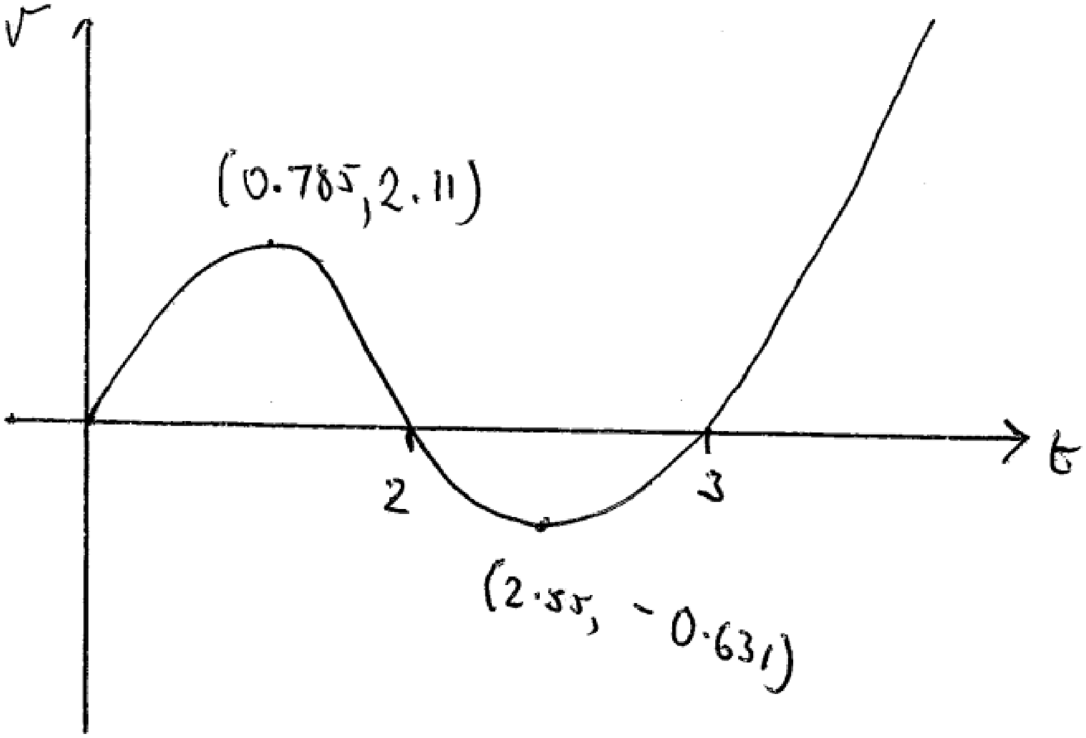
e. A second particle, B, moving along the same line, has position x_B m, velocity $v_B\ \mathrm{ms}^{-1}$ and acceleration, $a_B\ \mathrm{ms}^{-2}$, where $a_B = -2v_B$ for $t \geqslant 0$. At $t = 0,\ x_B = 20$ and $v_B = -20$. [4]

   Find an expression for v_B in terms of t.

f. Find the value of t when the two particles meet. [6]

Markscheme

a.



A1A1A1

Note: Award ***A1*** for general shape, ***A1*** for correct maximum and minimum, ***A1*** for intercepts.

Note: Follow through applies to (b) and (c).

[3 marks]

b. $0 \leqslant t < 0.785, \ \left(\text{or } 0 \leqslant t < \frac{5-\sqrt{7}}{3}\right)$ ***A1***

(allow $t < 0.785$)

and $t > 2.55 \ \left(\text{or } t > \frac{5+\sqrt{7}}{3}\right)$ ***A1***

[2 marks]

c. $0 \leqslant t < 0.785, \ \left(\text{or } 0 \leqslant t < \frac{5-\sqrt{7}}{3}\right)$ ***A1***

(allow $t < 0.785$)

$2 < t < 2.55, \ \left(\text{or } 2 < t < \frac{5+\sqrt{7}}{3}\right)$ ***A1***

$t > 3$ ***A1***

[3 marks]

d. position of A: $x_A = \int t^3 - 5t^2 + 6t \text{ d}t$ ***(M1)***

$x_A = \frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 \quad (+c)$ ***A1***

when $t = 0, \ x_A = 0$, so $c = 0$ ***R1***

[3 marks]

e. $\frac{\text{d}v_B}{\text{d}t} = -2v_B \Rightarrow \int \frac{1}{v_B}\text{d}v_B = \int -2\text{d}t$ ***(M1)***

$\ln|v_B| = -2t + c$ ***(A1)***

$v_B = Ae^{-2t}$ ***(M1)***

$v_B = -20$ when $t = 0$ so $v_B = -20e^{-2t}$ ***A1***

[4 marks]

f. $x_B = 10e^{-2t}(+c)$ *(M1)(A1)*

$x_B = 20$ when $t = 0$ so $x_B = 10e^{-2t} + 10$ *(M1)A1*

meet when $\frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 = 10e^{-2t} + 10$ *(M1)*

$t = 4.41(290\ldots)$ *A1*

[6 marks]

Examiners report

a. Part (a) was generally well done, although correct accuracy was often a problem.

b. Parts (b) and (c) were also generally quite well done.

c. Parts (b) and (c) were also generally quite well done.

d. A variety of approaches were seen in part (d) and many candidates were able to obtain at least 2 out of 3. A number missed to consider the $+c$, thereby losing the last mark.

e. Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

f. Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

Consider the expression $f(x) = \tan\left(x + \frac{\pi}{4}\right)\cot\left(\frac{\pi}{4} - x\right)$.

The expression $f(x)$ can be written as $g(t)$ where $t = \tan x$.

Let α, β be the roots of $g(t) = k$, where $0 < k < 1$.

a.i. Sketch the graph of $y = f(x)$ for $-\frac{5\pi}{8} \leqslant x \leqslant \frac{\pi}{8}$. [2]

a.ii. With reference to your graph, explain why f is a function on the given domain. [1]

a.iii. Explain why f has no inverse on the given domain. [1]

a.iv. Explain why f is not a function for $-\frac{3\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$. [1]

b. Show that $g(t) = \left(\frac{1+t}{1-t}\right)^2$. [3]

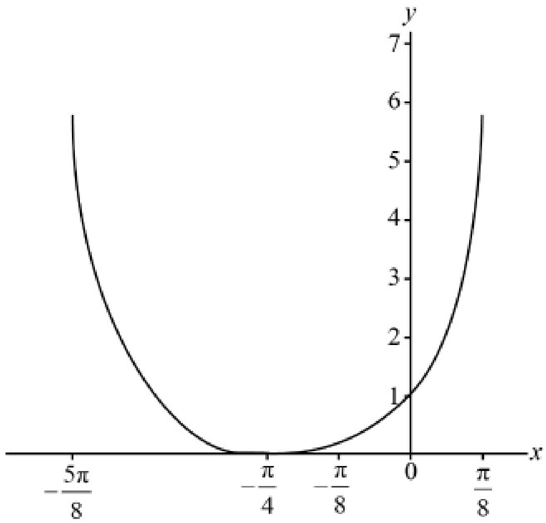
c. Sketch the graph of $y = g(t)$ for $t \leq 0$. Give the coordinates of any intercepts and the equations of any asymptotes. [3]

d.i. Find α and β in terms of k. [5]

d.ii.Show that $\alpha + \beta < -2$. [2]

Markscheme

a.i.



A1A1

A1 for correct concavity, many to one graph, symmetrical about the midpoint of the domain and with two axes intercepts.

Note: Axes intercepts and scales not required.

A1 for correct domain

[2 marks]

a.ii.for each value of x there is a unique value of $f(x)$ ***A1***

Note: Accept "passes the vertical line test" or equivalent.

[1 mark]

a.iii.no inverse because the function fails the horizontal line test or equivalent ***R1***

Note: No **FT** if the graph is in degrees (one-to-one).

[1 mark]

a.iv.the expression is not valid at either of $x = \frac{\pi}{4}\ \left(\text{or} - \frac{3\pi}{4}\right)$ ***R1***

[1 mark]

b. **METHOD 1**

$f(x) = \frac{\tan\left(x+\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}-x\right)}$ ***M1***

$= \frac{\frac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}}{\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$ ***M1A1***

$= \left(\frac{1+t}{1-t}\right)^2$ ***AG***

METHOD 2

$f(x) = \tan\left(x+\frac{\pi}{4}\right)\tan\left(\frac{\pi}{2}-\frac{\pi}{4}+x\right)$ ***(M1)***

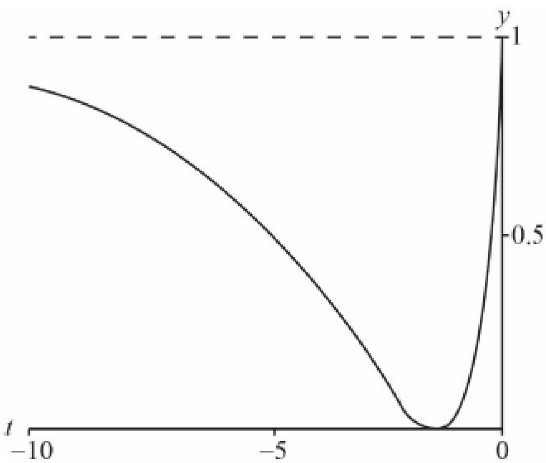
$= \tan^2\left(x+\frac{\pi}{4}\right)$ ***A1***

$g(t) = \left(\frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}}\right)^2$ ***A1***

$= \left(\frac{1+t}{1-t}\right)^2$ ***AG***

[3 marks]

c.



for $t \le 0$, correct concavity with two axes intercepts and with asymptote $y = 1$ ***A1***

t intercept at (–1, 0) ***A1***

y intercept at (0, 1) ***A1***

[3 marks]

d.i. **METHOD 1**

α, β satisfy $\frac{(1+t)^2}{(1-t)^2} = k$ ***M1***

$1 + t^2 + 2t = k\left(1 + t^2 - 2t\right)$ ***A1***

$(k-1)\,t^2 - 2\,(k+1)\,t + (k-1) = 0$ ***A1***

attempt at using quadratic formula ***M1***

$\alpha, \beta = \frac{k+1 \pm 2\sqrt{k}}{k-1}$ or equivalent ***A1***

METHOD 2

α, β satisfy $\frac{1+t}{1-t} = (\pm)\sqrt{k}$ ***M1***

$t + \sqrt{k}t = \sqrt{k} - 1$ ***M1***

$t = \frac{\sqrt{k}-1}{\sqrt{k}+1}$ (or equivalent) ***A1***

$t - \sqrt{k}t = -\left(\sqrt{k} + 1\right)$ ***M1***

$t = \frac{\sqrt{k}+1}{\sqrt{k}-1}$ (or equivalent) ***A1***

so for *eg*, $\alpha = \frac{\sqrt{k}-1}{\sqrt{k}+1}, \beta = \frac{\sqrt{k}+1}{\sqrt{k}-1}$

[5 marks]

d.ii $\alpha + \beta = 2\frac{(k+1)}{(k-1)}\left(= -2\frac{(1+k)}{(1-k)}\right)$ ***A1***

since $1 + k > 1 - k$ ***R1***

$\alpha + \beta < -2$ ***AG***

Note: Accept a valid graphical reasoning.

[2 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
a.iii. [N/A]
a.iv. [N/A]
b. [N/A]
c. [N/A]
d.i. [N/A]
d.ii. [N/A]

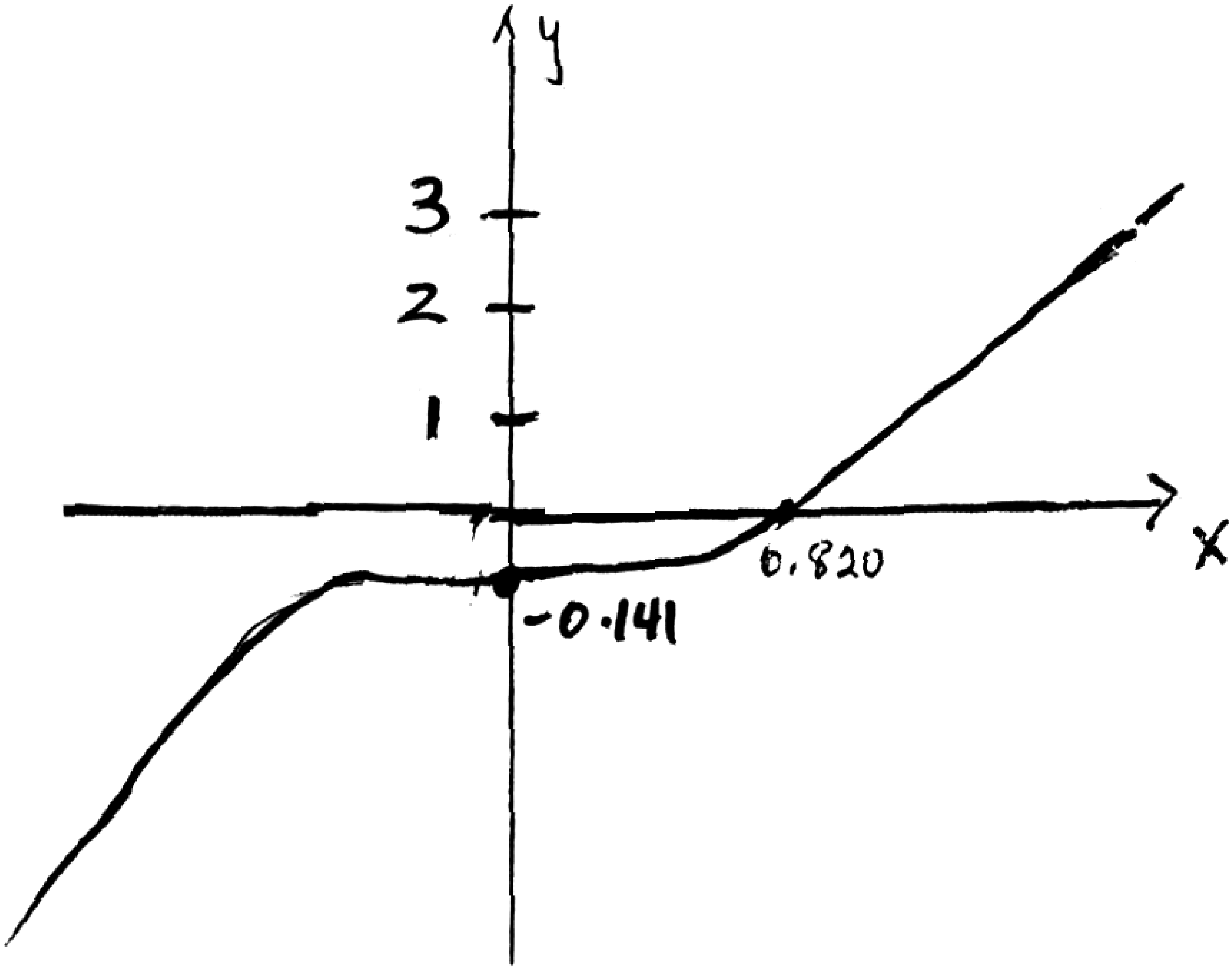
Consider the graph of $y = x + \sin(x - 3), \ -\pi \leqslant x \leqslant \pi$.

a. Sketch the graph, clearly labelling the x and y intercepts with their values. [3]

b. Find the area of the region bounded by the graph and the x and y axes. [2]

Markscheme

a.



A1A1A1

Note: Award ***A1*** for shape,

A1 for *x*-intercept is 0.820, accept $\sin(-3)$ or $-\sin(3)$

A1 for *y*-intercept is −0.141.

[3 marks]

b. $A = \int_0^{0.8202} |x + \sin(x - 3)|\, \mathrm{d}x \approx 0.0816$ sq units ***(M1)A1***

[2 marks]

Examiners report

a. Many candidates attempted this question successfully. In (a), however, a large number of candidates did not use the zoom feature of the GDC to draw an accurate sketch of the given function. In (b), some candidates used the domain as the limits of the integral. Other candidates did not take the absolute value of the integral.

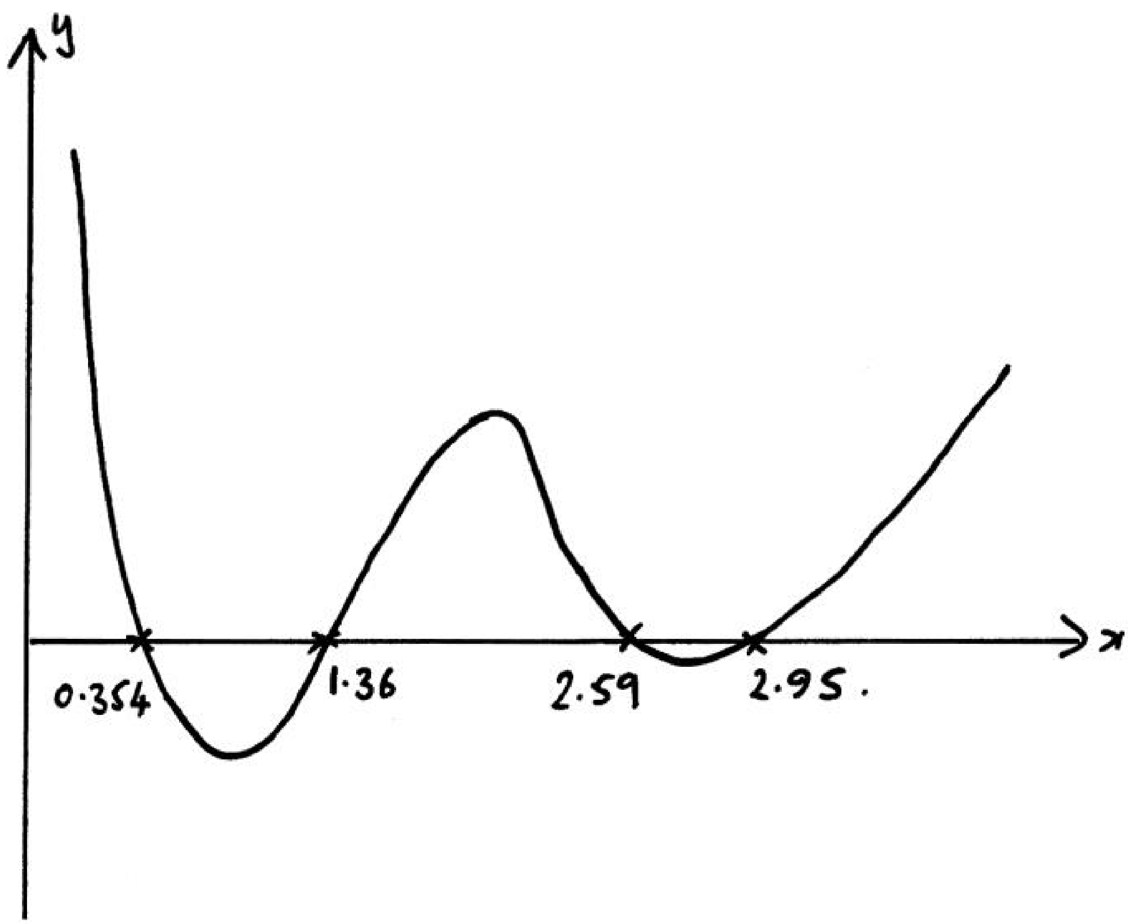
b. Many candidates attempted this question successfully. In (a), however, a large number of candidates did not use the zoom feature of the GDC to draw an accurate sketch of the given function. In (b), some candidates used the domain as the limits of the integral. Other candidates did not take the absolute value of the integral.

(a) Sketch the curve $y = |\ln x| - |\cos x| - 0.1$, $0 < x < 4$ showing clearly the coordinates of the points of intersection with the x-axis and the coordinates of any local maxima and minima.

(b) Find the values of x for which $|\ln x| > |\cos x| + 0.1$, $0 < x < 4$.

Markscheme

(a)



A1

Note: Award ***A1*** for shape.

x-intercepts 0.354, 1.36, 2.59, 2.95 ***A2***

Note: Award ***A1*** for three correct, ***A0*** otherwise.

maximum = (1.57, 0.352) = $\left(\frac{\pi}{2}, 0.352\right)$ ***A1***

minimum = (1, – 0.640) and (2.77, – 0.0129) ***A1***

(b) $0 < x < 0.354,\ 1.36 < x < 2.59,\ 2.95 < x < 4$ ***A2***

Note: Award ***A1*** if two correct regions given.

[7 marks]

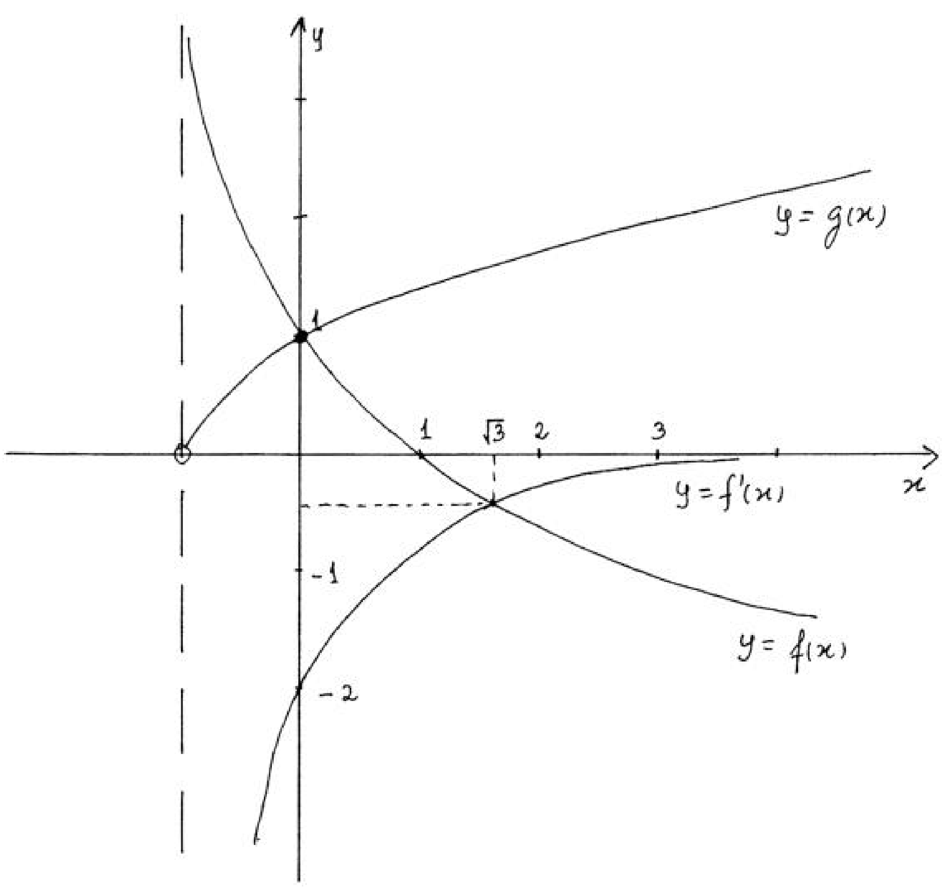
Examiners report

Solutions to this question were extremely disappointing with many candidates doing the sketch in degree mode instead of radian mode. The two adjacent intercepts at 2.59 and 2.95 were often missed due to an unsatisfactory window. Some sketches were so small that a magnifying glass was required to read some of the numbers; candidates would be well advised to draw sketches large enough to be easily read.

Let $f(x) = \frac{1-x}{1+x}$ and $g(x) = \sqrt{x+1}$, $x > -1$.

Find the set of values of x for which $f'(x) \leqslant f(x) \leqslant g(x)$.

Markscheme



$f'(x) = \frac{-2}{(1+x)^2}$ ***M1A1***

Note: Alternatively, award ***M1A1*** for correct sketch of the derivative.

find at least one point of intersection of graphs ***(M1)***

$y = f(x)$ and $y = f'(x)$ for $x = \sqrt{3}$ or 1.73 ***(A1)***

$y = f(x)$ and $y = g(x)$ for $x = 0$ ***(A1)***

forming inequality $0 \leqslant x \leqslant \sqrt{3}$ (or $0 \leqslant x \leqslant 1.73$) ***A1A1*** ***N4***

Note: Award ***A1*** for correct limits and ***A1*** for correct inequalities.

[7 marks]

Examiners report

Most students were able to find the derived function correctly, although attempts to solve the inequality algebraically were often unsuccessful.

This was a question where students prepared in good use of GDC were able to easily obtain good marks.

Let $f(x) = \frac{4-x^2}{4-\sqrt{x}}$.

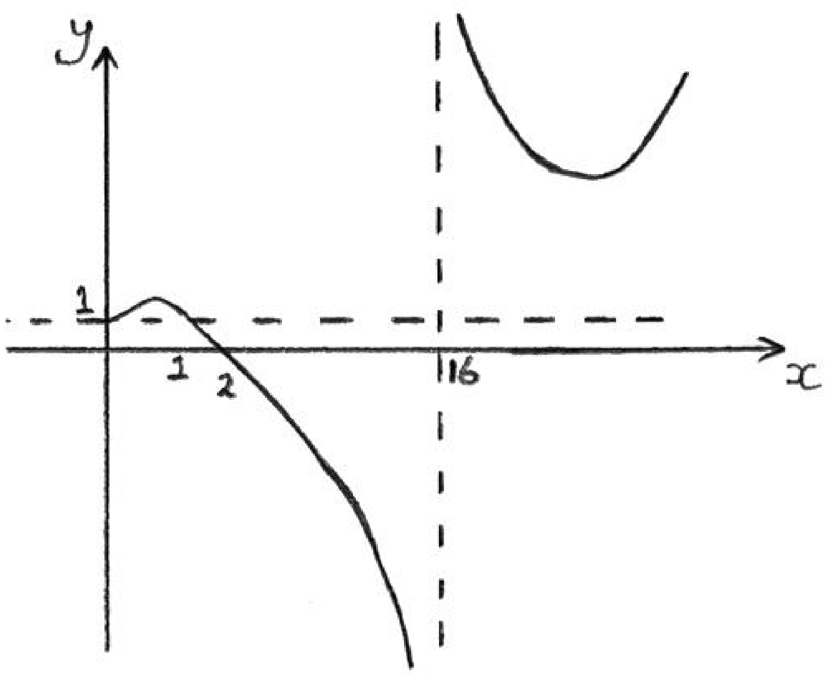
(a) State the largest possible domain for *f*.

(b) Solve the inequality $f(x) \geqslant 1$.

Markscheme

(a) $x \geqslant 0$ and $x \neq 16$ ***A1A1***

(b)



graph not to scale

finding crossing points ***(M1)***

e.g. $4 - x^2 = 4 - \sqrt{x}$

$x = 0$ or $x = 1$ ***(A1)***

$0 \leqslant x \leqslant 1$ or $x > 16$ ***A1A1***

Note: Award ***M1A1A1A0*** for solving the inequality only for the case $x < 16$.

[6 marks]

Examiners report

Most students were able to obtain partial marks, but there were very few completely correct answers.

Consider the function g , where $g(x) = \frac{3x}{5+x^2}$.

(a) Given that the domain of g is $x \geqslant a$, find the least value of a such that g has an inverse function.

(b) On the same set of axes, sketch

(i) the graph of g for this value of a ;

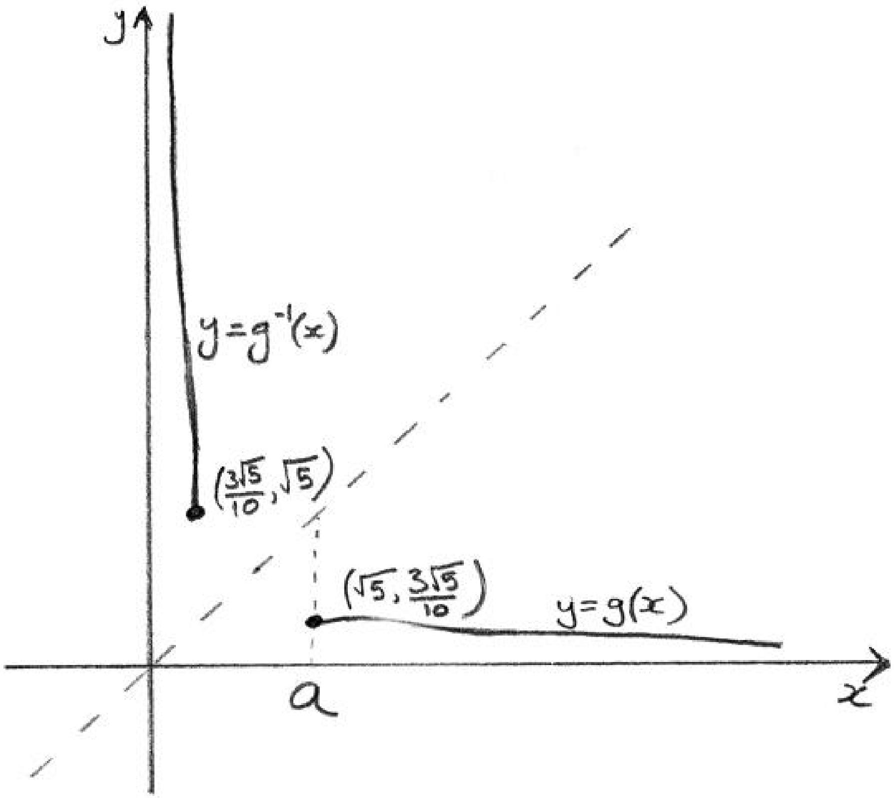
(ii) the corresponding inverse, g^{-1} .

(c) Find an expression for $g^{-1}(x)$.

Markscheme

(a) $a = 2.24$ $\sqrt{5}$ ***A1***

(b) (i)



A2

Note: Award ***A1*** for end point

A1 for its asymptote.

(ii) sketch of g^{-1} (see above) ***A2***

Note: Award ***A1*** for end point

A1 for its asymptote.

(c) $y = \frac{3x}{5+x^2} \Rightarrow yx^2 - 3x + 5y = 0$ ***M1***

$\Rightarrow x = \frac{3 \pm \sqrt{9-20y^2}}{2y}$ ***A1***

$g^{-1}(x) = \frac{3 \pm \sqrt{9-20x^2}}{2x}$ ***A1***

[8 marks]

Examiners report

Very few completely correct answers were given to this question. Many students found a to be 0 and many failed to provide adequate sketches.

There were very few correct answers to part (c) although many students were able to obtain partial marks.

A particle moves in a straight line with velocity *v* metres per second. At any time *t* seconds, $0 \leqslant t < \frac{3\pi}{4}$, the velocity is given by the differential equation $\frac{dv}{dt} + v^2 + 1 = 0$.

It is also given that *v* = 1 when *t* = 0 .

a. Find an expression for v in terms of t . [7]

b. Sketch the graph of v against t , clearly showing the coordinates of any intercepts, and the equations of any asymptotes. [3]

c. (i) Write down the time T at which the velocity is zero. [3]

(ii) Find the distance travelled in the interval $[0, T]$.

d. Find an expression for s , the displacement, in terms of t , given that $s = 0$ when $t = 0$. [5]

e. Hence, or otherwise, show that $s = \frac{1}{2}\ln\frac{2}{1+v^2}$. [4]

Markscheme

a. $\frac{\mathrm{d}v}{\mathrm{d}t} = -v^2 - 1$

attempt to separate the variables ***M1***

$\int \frac{1}{1+v^2}\mathrm{d}v = \int -1\mathrm{d}t$ ***A1***

$\arctan v = -t + k$ ***A1A1***

Note: Do not penalize the lack of constant at this stage.

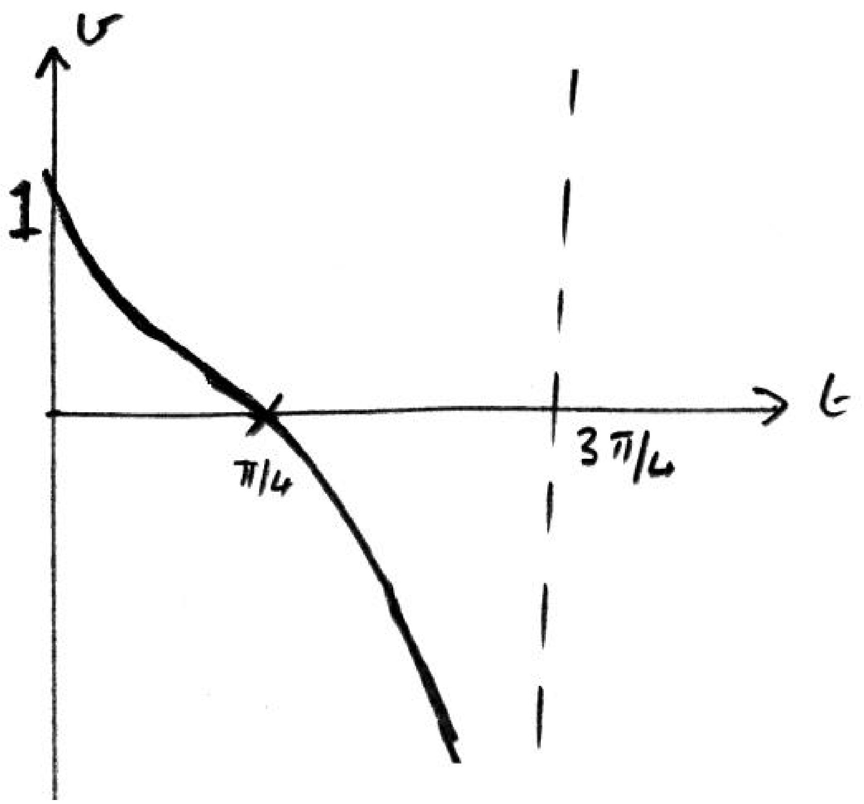
when $t = 0$, $v = 1$ ***M1***

$\Rightarrow k = \arctan 1 = \left(\frac{\pi}{4}\right) = (45^\circ)$ ***A1***

$\Rightarrow v = \tan\left(\frac{\pi}{4} - t\right)$ ***A1***

[7 marks]

b.



A1A1A1

Note: Award ***A1*** for general shape,
A1 for asymptote,
A1 for correct t and v intercept.

Note: Do not penalise if a larger domain is used.

[3 marks]

c. (i) $T = \frac{\pi}{4}$ ***A1***

(ii) area under curve $= \int_0^{\frac{\pi}{4}} \tan\left(\frac{\pi}{4} - t\right)\mathrm{d}t$ ***(M1)***

$= 0.347 \left(= \frac{1}{2}\ln 2\right)$ ***A1***

[3 marks]

d. $v = \tan\left(\frac{\pi}{4} - t\right)$

$s = \int \tan\left(\frac{\pi}{4} - t\right) \mathrm{d}t$ ***M1***

$\int \frac{\sin\left(\frac{\pi}{4} - t\right)}{\cos\left(\frac{\pi}{4} - t\right)} \mathrm{d}t$ ***(M1)***

$= \ln \cos\left(\frac{\pi}{4} - t\right) + k$ ***A1***

when $t = 0,\ s = 0$

$k = -\ln \cos \frac{\pi}{4}$ ***A1***

$s = \ln \cos\left(\frac{\pi}{4} - t\right) - \ln \cos \frac{\pi}{4} \left(= \ln\left[\sqrt{2} \cos\left(\frac{\pi}{4} - t\right)\right]\right)$ ***A1***

[5 marks]

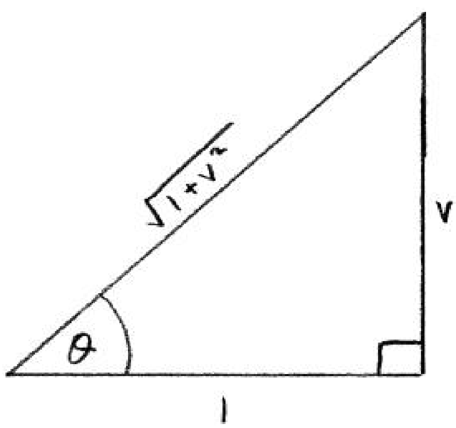
e. **METHOD 1**

$\frac{\pi}{4} - t = \arctan v$ ***M1***

$t = \frac{\pi}{4} - \arctan v$

$s = \ln\left[\sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{4} + \arctan v\right)\right]$

$s = \ln\left[\sqrt{2} \cos(\arctan v)\right]$ ***M1A1***



$s = \ln\left[\sqrt{2} \cos\left(\arccos \frac{1}{\sqrt{1+v^2}}\right)\right]$ ***A1***

$= \ln \frac{\sqrt{2}}{\sqrt{1+v^2}}$

$= \frac{1}{2}\ln \frac{2}{1+v^2}$ ***AG***

METHOD 2

$s = \ln \cos\left(\frac{\pi}{4} - t\right) - \ln \cos \frac{\pi}{4}$

$= -\ln \sec\left(\frac{\pi}{4} - t\right) - \ln \cos \frac{\pi}{4}$ ***M1***

$= -\ln \sqrt{1 + \tan^2\left(\frac{\pi}{4} - t\right)} - \ln \cos \frac{\pi}{4}$ ***M1***

$= -\ln \sqrt{1 + v^2} - \ln \cos \frac{\pi}{4}$ ***A1***

$= \ln \frac{1}{\sqrt{1+v^2}} + \ln \sqrt{2}$ ***A1***

$= \frac{1}{2}\ln \frac{2}{1+v^2}$ ***AG***

METHOD 3

$v\frac{dv}{ds} = -v^2 - 1$ ***M1***

$\int \frac{v}{v^2+1} dv = -\int 1 ds$ ***M1***

$\frac{1}{2}\ln(v^2 + 1) = -s + k$ ***A1***

when $s = 0,\ t = 0 \Rightarrow v = 1$

$\Rightarrow k = \frac{1}{2}\ln 2$ ***A1***

$\Rightarrow s = \frac{1}{2}\ln \frac{2}{1+v^2}$ ***AG***

[4 marks]

Examiners report

a. This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

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The function *f* is defined by

$$f(x) = (x^3 + 6x^2 + 3x - 10)^{\frac{1}{2}},\ \text{for } x \in D,$$

where $D \subseteq \mathbb{R}$ is the greatest possible domain of *f*.

(a) Find the roots of $f(x) = 0$.

(b) Hence specify the set D.

(c) Find the coordinates of the local maximum on the graph $y = f(x)$.

(d) Solve the equation $f(x) = 3$.

(e) Sketch the graph of $|y| = f(x)$, for $x \in D$.

(f) Find the area of the region completely enclosed by the graph of $|y| = f(x)$

Markscheme

(a) solving to obtain one root: 1, – 2 or – 5 ***A1***

obtain other roots ***A1***

[2 marks]

(b) $D = x \in [-5,\ -2] \cup [1,\ \infty)$ (or equivalent) ***M1A1***

Note: ***M1*** is for 1 finite and 1 infinite interval.

[2 marks]

(c) coordinates of local maximum $-3.73 - 2 - \sqrt{3},\ 3.22\sqrt{6\sqrt{3}}$ ***A1A1***

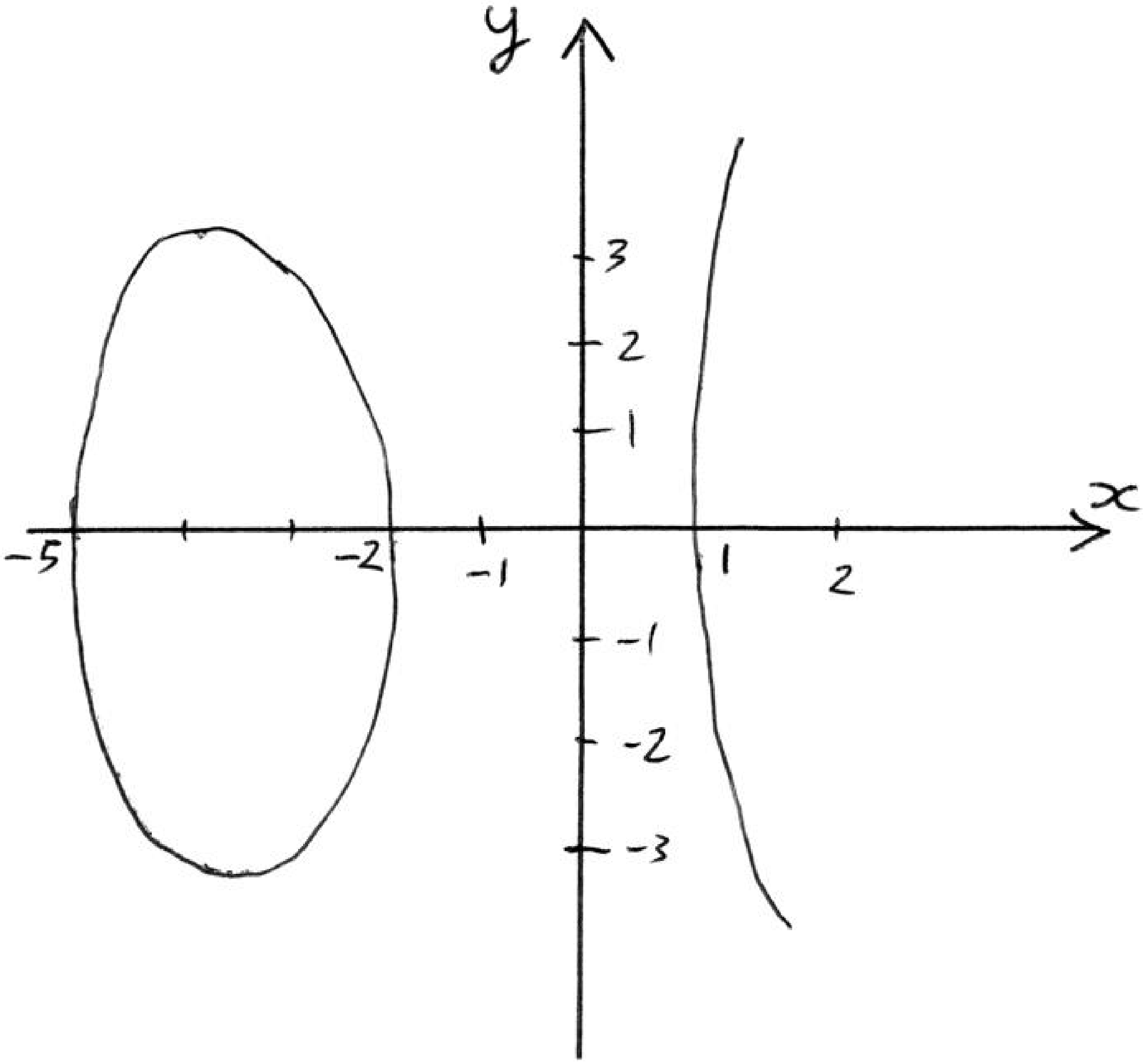
[2 marks]

(d) use GDC to obtain one root: 1.41, – 3.18 or – 4.23 ***A1***

obtain other roots ***A1***

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for shape, ***A1*** for max and for min clearly in correct places, ***A1*** for all intercepts.

Award ***A1A0A0*** if only the complete top half is shown.

[3 marks]

(f) required area is twice that of $y = f(x)$ between – 5 and – 2 ***M1A1***

answer 14.9 ***A1*** ***N3***

Note: Award ***M1A0A0*** for $\int_{-5}^{-2} f(x)\mathrm{d}x = 7.47\ldots$ or ***N1*** for 7.47.

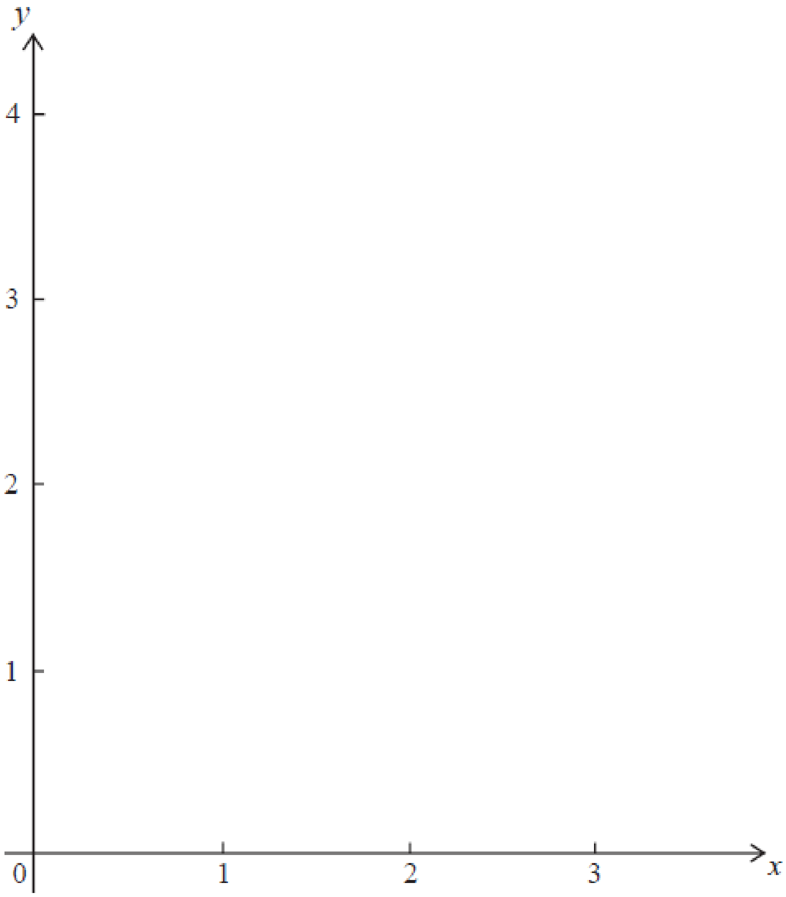
[3 marks]

Total [14 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. The main difficulty was sketching the graph and this meant that the last part was not well answered.

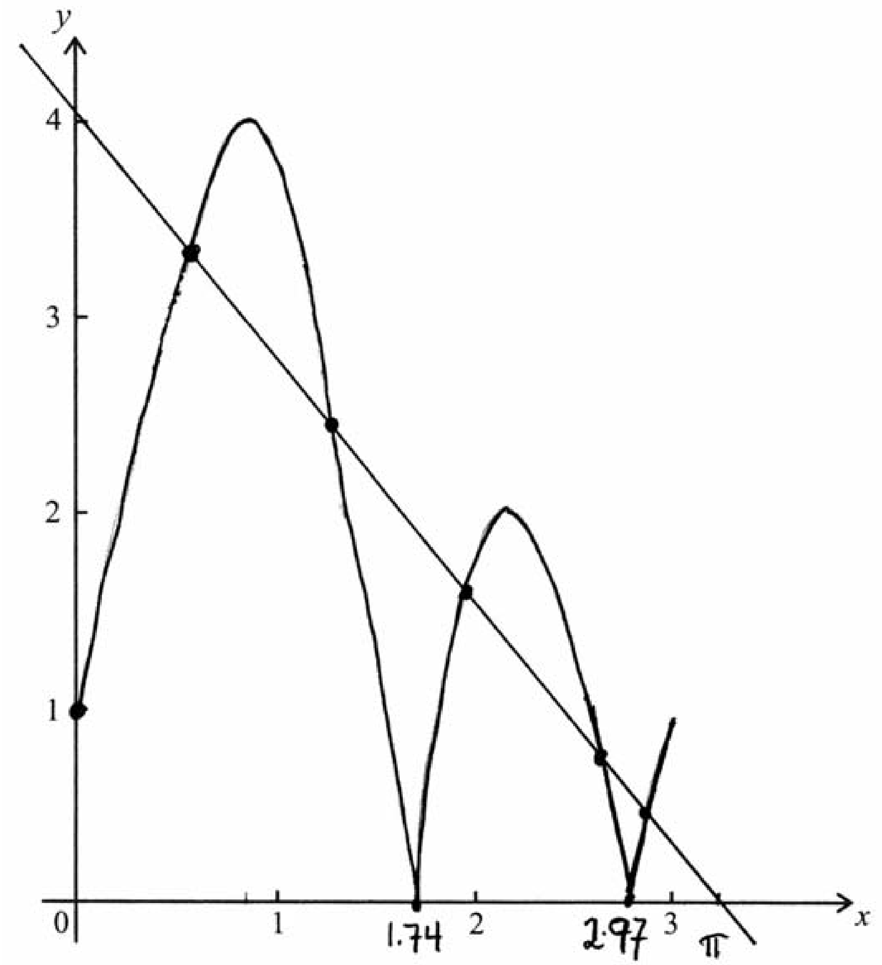
(a) Sketch the curve $f(x) = |1 + 3\sin(2x)|$, for $0 \leqslant x \leqslant \pi$. Write down on the graph the values of the x and y intercepts.



(b) By adding **one** suitable line to your sketch, find the number of solutions to the equation $\pi f(x) = 4(\pi - x)$.

Markscheme

(a)



A1A1A1A1

Note: Award ***A1*** for y-intercept

A1A1 for *x*-intercepts

A1 for shape

(b) correct line ***A1***

5 solutions ***A1***

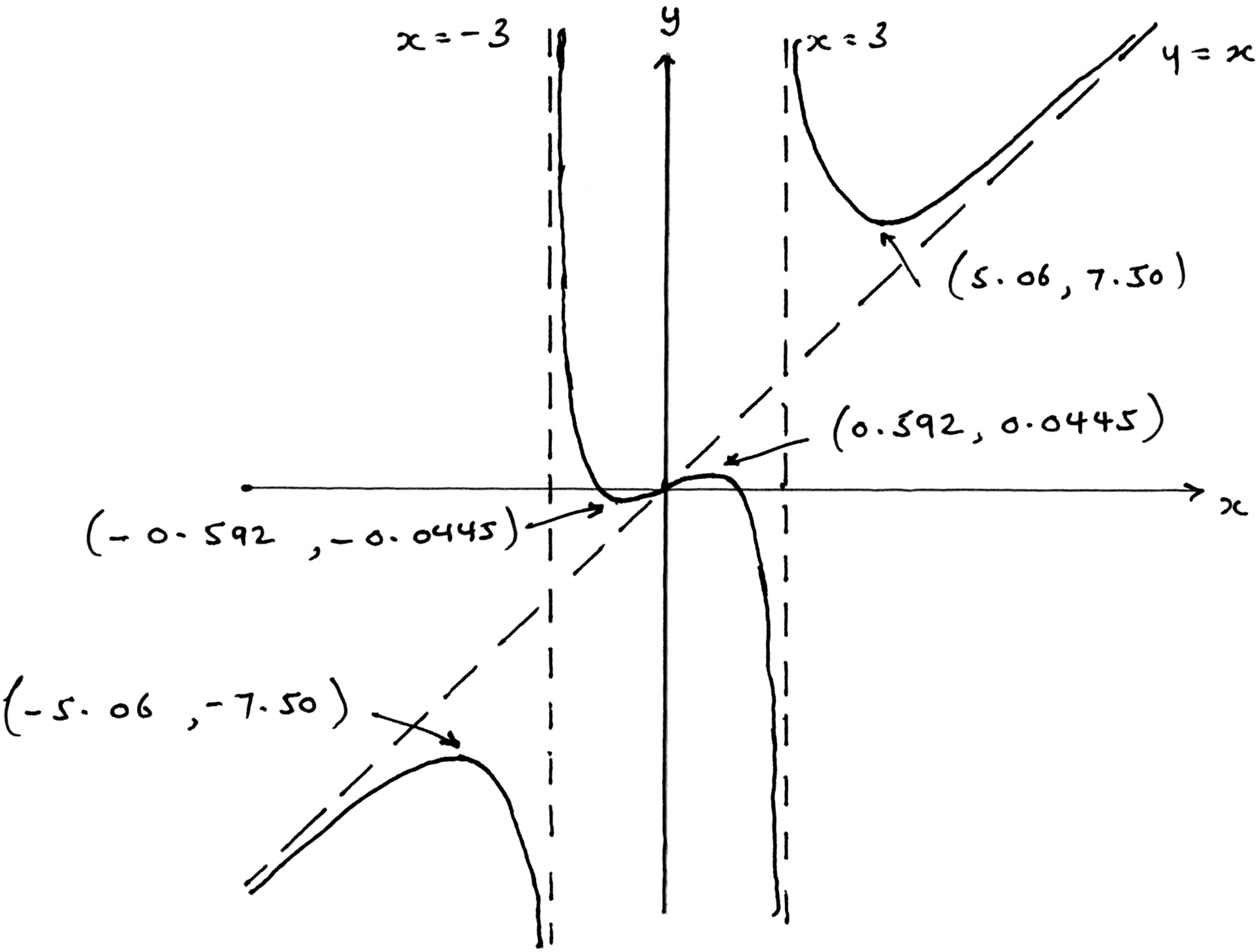
[6 marks]

Examiners report

Part (a) was well executed by the majority of candidates. Most candidates had the correct graph with the correct x and y intercepts. For part (b), some candidates had the straight line intersect the *x*-axis at 3 rather than at π , and hence did not observe that there were 5 points of intersection.

Sketch the graph of $f(x) = x + \frac{8x}{x^2-9}$. Clearly mark the coordinates of the two maximum points and the two minimum points. Clearly mark and state the equations of the vertical asymptotes and the oblique asymptote.

Markscheme



M1A1A1A1A1A1A1

Note: Award ***A1*** for both vertical asymptotes correct,

M1 for recognizing that there are two turning points near the origin,

A1 for both turning points near the origin correct, (only this ***A*** mark is dependent on the ***M*** mark)

A1 for the other pair of turning points correct,

A1 for correct positioning of the oblique asymptote,

A1 for correct equation of the oblique asymptote,

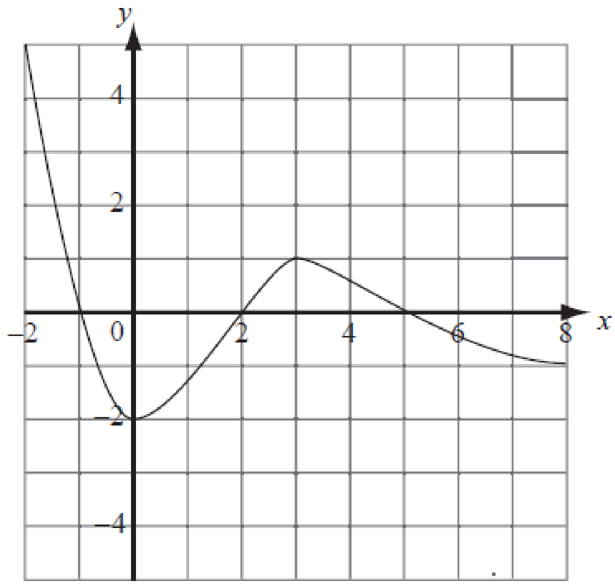
A1 for correct asymptotic behaviour in all sections.

[7 marks]

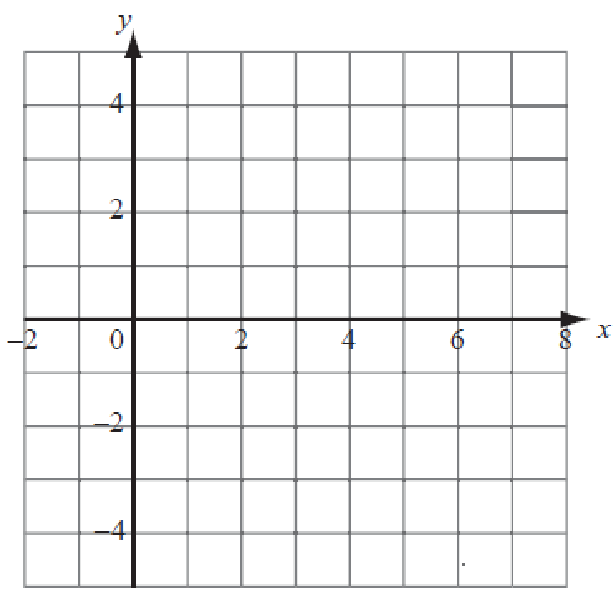
Examiners report

This question was generally well done, except for the behaviour near the origin. The questions alerted candidates to the existence of four turning points and an oblique asymptote, but not all reported back on this information.

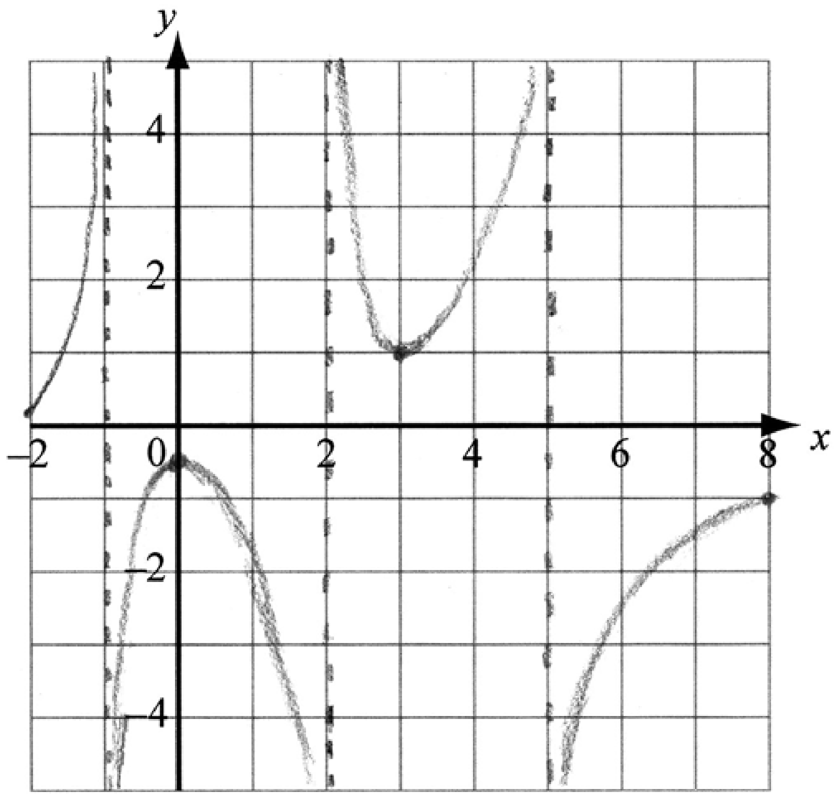
The graph of $y = f(x)$ for $-2 \leqslant x \leqslant 8$ is shown.



On the set of axes provided, sketch the graph of $y = \frac{1}{f(x)}$, clearly showing any asymptotes and indicating the coordinates of any local maxima or minima.



Markscheme



A1A1A1A1A1

Notes: Award ***A1*** for vertical asymptotes at $x = -1$, $x = 2$ and $x = 5$.

A1 for $x \to -2,\ \frac{1}{f(x)} \to 0^+$

A1 for $x \to 8,\ \frac{1}{f(x)} \to -1$

A1 for local maximum at $\left(0, -\frac{1}{2}\right)$ (branch containing local max. must be present)

A1 for local minimum at (3, 1) (branch containing local min. must be present)

In each branch, correct asymptotic behaviour must be displayed to obtain the ***A1***.

Disregard any stated horizontal asymptotes such as $y = 0$ or $y = -1$.

[5 marks]

Examiners report

A large number of candidates had difficulty graphing the reciprocal function. Most candidates were able to locate the vertical asymptotes but experienced difficulties graphing the four constituent branches. A common error was to specify incorrect coordinates of the local maximum *i.e.* (0,–1) or (0,–2) instead of $\left(0, -\frac{1}{2}\right)$. A few candidates attempted to sketch the inverse while others had difficulty using the scaled grid.